

M2-brane Perspective on $\mathcal{N} = 6$ Super Chern-Simons Theory at Level k

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Abstract

Recently, O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena (ABJM) proposed three-dimensional super Chern-Simons-matter theory, which at level k is supposed to describe the low energy limit of N M2-branes. For large N and k , but fixed 't Hooft coupling $\lambda = N/k$, it is dual to type IIA string theory on $AdS_4 \times \mathbb{CP}^3$. For large N but *finite* k , it is dual to M theory on $AdS_4 \times S^7/Z_k$. In this paper, relying on the second duality, we find exact giant magnon and single spike solutions of membrane configurations on $AdS_4 \times S^7/Z_k$ by reducing the system to the Neumann-Rosochatius integrable model. We derive the dispersion relations and their finite-size corrections with explicit dependence on the level k .

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1 Introduction

The *AdS/CFT* duality conjecture [1, 2, 3], has led to many interesting developments in understanding the correspondence between type IIB string theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ super Yang-Mills theory in four dimensions. Recently, new exciting field for investigations appeared, after the discovery of the new *AdS₄/CFT₃* duality [4]. The most promising candidate for description of this correspondence is the $\mathcal{N} = 6$ super Chern-Simons-matter theory proposed by ABJM in [5]. This theory at level k describes the low energy limit of N M2-branes probing a C^4/Z_k singularity. At large N , it is dual to M theory on $AdS_4 \times S^7/Z_k$. For large N and fixed ratio N/k , it also has a 't Hooft limit, which is dual to type IIA string theory on $AdS_4 \times \mathbb{CP}^3$.

After the appearance of [5], many related papers quickly followed [6]-[41], investigating different aspects of the ABJM theory. We would like to mention the discovery of the exact S -matrix, including the dressing phase [25], confirming the all-loop Bethe ansatz equations, conjectured in [22].

In this article, we propose an M2-brane viewpoint on the ABJM theory at finite level k , by considering membrane configurations with two angular momenta on $R_t \times S^7/Z_k$ subspace of $AdS_4 \times S^7/Z_k$ background, which exhibit similar properties of giant magnon (GM) [42], dyonic GM [43] and single spike (SS) [44] in string theory.

The paper is organized as follows. In section 2, we find an appropriate M2-brane embedding into $R_t \times S^7/Z_k$. In section 3, we show that there exists unique Neumann-Rosochatius (NR) integrable system, which describes membrane configurations with two angular momenta, for any finite value of the level k . In section 4, based on this NR approach, we give the corresponding M2-brane GM and SS solutions and the semiclassical energy-charge relations, including the finite-size effects. We conclude the paper with some comments in section 5.

2 Membranes on $AdS_4 \times S^7/Z_k$

Let us start with the following membrane action

$$S = \int d^3\xi \left\{ \frac{1}{4\lambda^0} \left[G_{00} - 2\lambda^j G_{0j} + \lambda^i \lambda^j G_{ij} - (2\lambda^0 T_2)^2 \det G_{ij} \right] + T_2 C_{012} \right\}, \quad (2.1)$$

where

$$\begin{aligned} G_{mn} &= g_{MN}(X)\partial_m X^M \partial_n X^N, \quad C_{012} = c_{MNP}(X)\partial_0 X^M \partial_1 X^N \partial_2 X^P, \\ \partial_m &= \partial/\partial\xi^m, \quad m = (0, i) = (0, 1, 2), \\ (\xi^0, \xi^1, \xi^2) &= (\tau, \sigma_1, \sigma_2), \quad M = (0, 1, \dots, 10), \end{aligned}$$

are the fields induced on the membrane worldvolume from the background metric g_{MN} and the background 3-form gauge field c_{MNP} , λ^m are Lagrange multipliers, $x^M = X^M(\xi)$ are the membrane embedding coordinates, and T_2 is its tension. As shown in [45], the above action is classically equivalent to the Nambu-Goto and Polyakov type actions. In addition, the action (2.1) gives a *unified* description for the tensile and tensionless membranes.

The equations of motion for the Lagrange multipliers λ^m generate the independent constraints only

$$G_{00} - 2\lambda^j G_{0j} + \lambda^i \lambda^j G_{ij} + (2\lambda^0 T_2)^2 \det G_{ij} = 0, \quad (2.2)$$

$$G_{0j} - \lambda^i G_{ij} = 0. \quad (2.3)$$

Further on, we will work in diagonal worldvolume gauge $\lambda^i = 0$, in which the action (2.1) and the constraints (2.2), (2.3) simplify to

$$S_M = \int d^3\xi \mathcal{L}_M = \int d^3\xi \left\{ \frac{1}{4\lambda^0} \left[G_{00} - (2\lambda^0 T_2)^2 \det G_{ij} \right] + T_2 C_{012} \right\}, \quad (2.4)$$

$$G_{00} + (2\lambda^0 T_2)^2 \det G_{ij} = 0, \quad (2.5)$$

$$G_{0i} = 0. \quad (2.6)$$

Let us introduce the following complex coordinates on the S^7/Z_k subspace

$$\begin{aligned} z_1 &= \cos \psi \cos \frac{\theta_1}{2} e^{i\left[\frac{\varphi}{k} + \frac{1}{2}(\phi_1 + \phi_3)\right]}, & z_2 &= \cos \psi \sin \frac{\theta_1}{2} e^{i\left[\frac{\varphi}{k} - \frac{1}{2}(\phi_1 - \phi_3)\right]}, \\ z_3 &= \sin \psi \cos \frac{\theta_2}{2} e^{i\left[\frac{\varphi}{k} + \frac{1}{2}(\phi_2 - \phi_3)\right]}, & z_4 &= \sin \psi \sin \frac{\theta_2}{2} e^{i\left[\frac{\varphi}{k} - \frac{1}{2}(\phi_2 + \phi_3)\right]}. \end{aligned}$$

Obviously, they satisfy the relation

$$\sum_{a=1}^4 z_a \bar{z}_a \equiv 1.$$

Next, we compute the metric

$$ds_{S^7/Z_k}^2 = \sum_{a=1}^4 dz_a d\bar{z}_a = \frac{1}{k^2} (d\varphi + kA_1)^2 + ds_{\mathbb{CP}^3}^2,$$

where

$$\begin{aligned}
A_1 &= \frac{1}{2} [\cos^2 \psi \cos \theta_1 d\phi_1 + \sin^2 \psi \cos \theta_2 d\phi_2 + (\cos^2 \psi - \sin^2 \psi) d\phi_3], \\
ds_{\mathbb{CP}^3}^2 &= d\psi^2 + \sin^2 \psi \cos^2 \psi \left(\frac{1}{2} \cos \theta_1 d\phi_1 - \frac{1}{2} \cos \theta_2 d\phi_2 + d\phi_3 \right)^2 \\
&+ \frac{1}{4} \cos^2 \psi (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{1}{4} \sin^2 \psi (d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2). \tag{2.7}
\end{aligned}$$

The membrane embedding into $R_t \times S^7/Z_k$, appropriate for our purposes, is

$$X_0 = \frac{R}{2} t(\xi^m), \quad W_a = R r_a(\xi^m) e^{i\varphi_a(\xi^m)}, \quad a = (1, 2, 3, 4),$$

where t is the *AdS* time, r_a are real functions of ξ^m , while φ_a are the isometric coordinates on which the background metric does not depend. The four complex coordinates W_a are restricted by the real embedding condition

$$\sum_{a=1}^4 W_a \bar{W}_a = R^2, \quad \text{or} \quad \sum_{a=1}^4 r_a^2 = 1.$$

The coordinates r_a are connected to the initial coordinates, on which the background depends, in an obvious way.

For the embedding described above, the metric induced on the M2-brane worldvolume is given by

$$G_{mn} = \frac{R^2}{4} \left[-\partial_m t \partial_n t + 4 \sum_{a=1}^4 (\partial_m r_a \partial_n r_a + r_a^2 \partial_m \varphi_a \partial_n \varphi_a) \right]. \tag{2.8}$$

Correspondingly, the membrane Lagrangian becomes

$$\mathcal{L} = \mathcal{L}_M + \Lambda \left(\sum_{a=1}^4 r_a^2 - 1 \right),$$

where Λ is a Lagrange multiplier.

3 NR integrable system for M2-branes on $R_t \times S^7/Z_k$

Let us consider the following particular case of the above membrane embedding [46]

$$\begin{aligned}
X_0 &= \frac{R}{2} \kappa \tau, \quad W_a = R r_a(\xi, \eta) e^{i[\omega_a \tau + \mu_a(\xi, \eta)]}, \\
\xi &= \alpha \sigma_1 + \beta \tau, \quad \eta = \gamma \sigma_2 + \delta \tau,
\end{aligned} \tag{3.1}$$

which implies

$$t = \kappa\tau, \quad \varphi_a(\xi^m) = \varphi_a(\tau, \sigma_1, \sigma_2) = \omega_a\tau + \mu_a(\xi, \eta). \quad (3.2)$$

Here $\kappa, \omega_a, \alpha, \beta, \gamma, \delta$ are parameters. For this ansatz, the membrane Lagrangian takes the form ($\partial_\xi = \partial/\partial\xi, \partial_\eta = \partial/\partial\eta$)

$$\begin{aligned} \mathcal{L} = & -\frac{R^2}{4\lambda^0} \left\{ (2\lambda^0 T_2 R \alpha \gamma)^2 \sum_{a < b=1}^4 [(\partial_\xi r_a \partial_\eta r_b - \partial_\eta r_a \partial_\xi r_b)^2 \right. \\ & + (\partial_\xi r_a \partial_\eta \mu_b - \partial_\eta r_a \partial_\xi \mu_b)^2 r_b^2 + (\partial_\xi \mu_a \partial_\eta r_b - \partial_\eta \mu_a \partial_\xi r_b)^2 r_a^2 \\ & + (\partial_\xi \mu_a \partial_\eta \mu_b - \partial_\eta \mu_a \partial_\xi \mu_b)^2 r_a^2 r_b^2] \\ & + \sum_{a=1}^4 \left[(2\lambda^0 T_2 R \alpha \gamma)^2 (\partial_\xi r_a \partial_\eta \mu_a - \partial_\eta r_a \partial_\xi \mu_a)^2 - (\beta \partial_\xi \mu_a + \delta \partial_\eta \mu_a + \omega_a)^2 \right] r_a^2 \\ & \left. - \sum_{a=1}^4 (\beta \partial_\xi r_a + \delta \partial_\eta r_a)^2 + (\kappa/2)^2 \right\} + \Lambda \left(\sum_{a=1}^4 r_a^2 - 1 \right). \end{aligned}$$

We have found a set of sufficient conditions, which reduce the above Lagrangian to the NR one. First of all, two of the angles φ_a should be set to zero. The corresponding r_a coordinates must depend only on η in a specific way. The remaining variables r_a and μ_a can depend only on ξ^1 . In principle, there are six such possibilities. How they are realized for the $R_t \times S^7/Z_k$ background, we will discuss in the next section. Here, we will work out the following example

$$\begin{aligned} r_1 = r_1(\xi), \quad r_2 = r_2(\xi), \quad \mu_1 = \mu_1(\xi), \quad \mu_2 = \mu_2(\xi), \\ r_3 = r_3(\eta) = r_0 \sin \eta, \quad r_4 = r_4(\eta) = r_0 \cos \eta, \quad r_0 < 1, \\ \varphi_3 = \varphi_4 = 0. \end{aligned} \quad (3.3)$$

For this choice, we receive (prime is used for $d/d\xi$)

$$\begin{aligned} \mathcal{L} = & -\frac{R^2}{4\lambda^0} \left\{ \sum_{a=1}^2 \left[(\tilde{A}^2 - \beta^2) r_a^2 + (\tilde{A}^2 - \beta^2) r_a^2 \left(\mu'_a - \frac{\beta \omega_a}{\tilde{A}^2 - \beta^2} \right)^2 - \frac{\tilde{A}^2}{\tilde{A}^2 - \beta^2} \omega_a^2 r_a^2 \right] \right. \\ & \left. + (\kappa/2)^2 - r_0^2 \delta^2 \right\} + \Lambda \left[\sum_{a=1}^2 r_a^2 - (1 - r_0^2) \right], \end{aligned}$$

where $\tilde{A}^2 \equiv (2\lambda^0 T_2 R \alpha \gamma r_0)^2$. Now we can integrate once the equations of motion for μ_a following from the above Lagrangian to get

$$\mu'_a = \frac{1}{\tilde{A}^2 - \beta^2} \left(\frac{C_a}{r_a^2} + \beta \omega_a \right), \quad (3.4)$$

¹Of course, the roles of ξ and η can be interchanged in this context.

where C_a are arbitrary constants. By using (3.4) in the equations of motion for $r_a(\xi)$, one finds that they can be obtained from the effective Lagrangian

$$L_{NR} = \sum_{a=1}^2 \left[(\tilde{A}^2 - \beta^2) r_a'^2 - \frac{1}{\tilde{A}^2 - \beta^2} \left(\frac{C_a^2}{r_a^2} + \tilde{A}^2 \omega_a^2 r_a^2 \right) \right] + \Lambda_M \left[\sum_{a=1}^2 r_a^2 - (1 - r_0^2) \right].$$

This Lagrangian, in full analogy with the string considerations [47, 48, 49], corresponds to particular case of the n -dimensional NR integrable system. For $C_a = 0$ one obtains the Neumann integrable model, which in the case at hand describes two-dimensional harmonic oscillator, constrained to a circle of radius $\sqrt{1 - r_0^2}$.

Let us consider the three constraints (2.5), (2.6) for the present case. For more close correspondence with the string case, we want the third one, $G_{02} = 0$, to be identically satisfied. To this end, since $G_{02} \sim r_0^2 \gamma \delta$, we set $\delta = 0$, i.e. $\eta = \gamma \sigma_2$. Then, the first two constraints give the conserved Hamiltonian H_{NR} and a relation between the parameters involved:

$$\begin{aligned} H_{NR} &= \sum_{a=1}^2 \left[(\tilde{A}^2 - \beta^2) r_a'^2 + \frac{1}{\tilde{A}^2 - \beta^2} \left(\frac{C_a^2}{r_a^2} + \tilde{A}^2 \omega_a^2 r_a^2 \right) \right] = \frac{\tilde{A}^2 + \beta^2}{\tilde{A}^2 - \beta^2} (\kappa/2)^2, \\ \sum_{a=1}^2 \omega_a C_a + \beta (\kappa/2)^2 &= 0. \end{aligned} \quad (3.5)$$

For closed membranes, r_a and μ_a must satisfy the following periodicity conditions

$$r_a(\xi + 2\pi\alpha, \eta + 2\pi\gamma) = r_a(\xi, \eta), \quad \mu_a(\xi + 2\pi\alpha, \eta + 2\pi\gamma) = \mu_a(\xi, \eta) + 2\pi n_a, \quad (3.6)$$

where n_a are integer winding numbers. In particular, γ is a non-zero integer.

Since the background metric does not depend on t and φ_a , the corresponding conserved quantities are the membrane energy E and four angular momenta J_a , defined by

$$E = - \int d^2\sigma \frac{\partial \mathcal{L}}{\partial(\partial_0 t)}, \quad J_a = \int d^2\sigma \frac{\partial \mathcal{L}}{\partial(\partial_0 \varphi_a)}, \quad a = 1, 2, 3, 4.$$

For our ansatz (3.3) $J_3 = J_4 = 0$. The energy and the other two angular momenta are given by

$$E = \frac{\pi R^2 \kappa}{4\lambda^0 \alpha} \int d\xi, \quad J_a = \frac{\pi R^2}{\lambda^0 \alpha (\tilde{A}^2 - \beta^2)} \int d\xi \left(\beta C_a + \tilde{A}^2 \omega_a r_a^2 \right), \quad a = 1, 2. \quad (3.7)$$

From here, by using the constraints (3.5), one obtains the energy-charge relation

$$\frac{4}{\tilde{A}^2 - \beta^2} \left[\tilde{A}^2 (1 - r_0^2) + \beta \sum_{a=1}^2 \frac{C_a}{\omega_a} \right] \frac{E}{\kappa} = \sum_{a=1}^2 \frac{J_a}{\omega_a}.$$

As usual, it is linear with respect to E and J_a before taking the semiclassical limit.

To identically satisfy the embedding condition

$$\sum_{a=1}^2 r_a^2 - (1 - r_0^2) = 0,$$

we set

$$r_1(\xi) = \sqrt{1 - r_0^2} \sin \theta(\xi), \quad r_2(\xi) = \sqrt{1 - r_0^2} \cos \theta(\xi).$$

Then from the conservation of the NR Hamiltonian (3.5) one finds

$$\theta' = \frac{\pm 1}{\tilde{A}^2 - \beta^2} \left[(\tilde{A}^2 + \beta^2) \tilde{\kappa}^2 - \frac{\tilde{C}_1^2}{\sin^2 \theta} - \frac{\tilde{C}_2^2}{\cos^2 \theta} - \tilde{A}^2 (\omega_1^2 \sin^2 \theta + \omega_2^2 \cos^2 \theta) \right]^{1/2}, \quad (3.8)$$

$$\sum_{a=1}^2 \omega_a \tilde{C}_a + \beta \tilde{\kappa}^2 = 0, \quad \tilde{\kappa}^2 = \frac{(\kappa/2)^2}{1 - r_0^2}, \quad \tilde{C}_a^2 = \frac{C_a^2}{(1 - r_0^2)^2}.$$

By replacing the solution for $\theta(\xi)$ received from (3.8) into (3.4), one obtains the solutions for μ_a :

$$\mu_1 = \frac{1}{\tilde{A}^2 - \beta^2} \left(\tilde{C}_1 \int \frac{d\xi}{\sin^2 \theta} + \beta \omega_1 \xi \right), \quad \mu_2 = \frac{1}{\tilde{A}^2 - \beta^2} \left(\tilde{C}_2 \int \frac{d\xi}{\cos^2 \theta} + \beta \omega_2 \xi \right). \quad (3.9)$$

The above analysis shows that the NR integrable models for membranes on $R_t \times S^7$ and $R_t \times S^7/Z_k$ are the same [21]. Therefore, we can use the results obtained in [21] for the present case. For convenience, the corresponding solutions and dispersion relations are given in the appendix.

4 M2-brane solutions on $R_t \times S^7/Z_k$

For our membrane embedding in $R_t \times S^7/Z_k$, the angular variables φ_a are related to the corresponding background coordinates as follows

$$\varphi_1 = \frac{\varphi}{k} + \frac{1}{2} (\phi_1 + \phi_3), \quad \varphi_2 = \frac{\varphi}{k} - \frac{1}{2} (\phi_1 - \phi_3),$$

$$\varphi_3 = \frac{\varphi}{k} + \frac{1}{2} (\phi_2 - \phi_3), \quad \varphi_4 = \frac{\varphi}{k} - \frac{1}{2} (\phi_2 + \phi_3).$$

As a consequence, for the angular momenta we have

$$\begin{aligned} J_{\varphi_1} &= \frac{J_\varphi}{k} + \frac{1}{2}(J_{\phi_1} + J_{\phi_3}), & J_{\varphi_2} &= \frac{J_\varphi}{k} - \frac{1}{2}(J_{\phi_1} - J_{\phi_3}), \\ J_{\varphi_3} &= \frac{J_\varphi}{k} + \frac{1}{2}(J_{\phi_2} - J_{\phi_3}), & J_{\varphi_4} &= \frac{J_\varphi}{k} - \frac{1}{2}(J_{\phi_2} + J_{\phi_3}). \end{aligned}$$

φ_a and J_{φ_a} satisfy the equalities

$$\sum_{a=1}^4 \varphi_a = \frac{4}{k}\varphi, \quad \sum_{a=1}^4 J_{\varphi_a} = \frac{4}{k}J_\varphi.$$

One of the conditions for the existence of NR description of the M2-brane dynamics is that two of the angles φ_a must be zero, which means that two of the four angular momenta J_{φ_a} vanish. The six possible cases are

$$\begin{aligned} \bullet \varphi_1 &= \phi_3 + \frac{\phi_1}{2} = \frac{2}{k}\varphi + \frac{\phi_1}{2}, & \varphi_2 &= \phi_3 - \frac{\phi_1}{2} = \frac{2}{k}\varphi - \frac{\phi_1}{2}, & \varphi_3 &= 0, & \varphi_4 &= 0; \\ \bullet \varphi_1 &= \phi_1 = \frac{2}{k}\varphi + \phi_3, & \varphi_3 &= \phi_2 = \frac{2}{k}\varphi - \phi_3, & \varphi_2 &= 0, & \varphi_4 &= 0; \\ \bullet \varphi_1 &= \phi_1 = \frac{2}{k}\varphi + \phi_3, & \varphi_4 &= -\phi_2 = \frac{2}{k}\varphi - \phi_3, & \varphi_2 &= 0, & \varphi_3 &= 0; \\ \bullet \varphi_2 &= -\phi_1 = \frac{2}{k}\varphi + \phi_3, & \varphi_3 &= \phi_2 = \frac{2}{k}\varphi - \phi_3, & \varphi_1 &= 0, & \varphi_4 &= 0; \\ \bullet \varphi_2 &= -\phi_1 = \frac{2}{k}\varphi + \phi_3, & \varphi_4 &= -\phi_2 = \frac{2}{k}\varphi - \phi_3, & \varphi_1 &= 0, & \varphi_3 &= 0; \\ \bullet \varphi_3 &= -\phi_3 + \frac{\phi_2}{2} = \frac{2}{k}\varphi + \frac{\phi_2}{2}, & \varphi_4 &= -\phi_3 - \frac{\phi_2}{2} = \frac{2}{k}\varphi - \frac{\phi_2}{2}, & \varphi_1 &= 0, & \varphi_2 &= 0. \end{aligned} \tag{4.1}$$

Here, ϕ_1 and ϕ_2 are the isometry angles on the two two-spheres inside \mathbb{CP}^3 , while ϕ_3 is isometry angle on the $U(1)$ fiber over $S^2 \times S^2$, as can be seen from (2.7).

From (4.1) it is clear that we have two alternative descriptions for φ_a . One is only in terms of the isometry angles on \mathbb{CP}^3 , and the other includes the eleventh coordinate φ . This is a consequence of our restriction to M2-brane configurations, which can be described by the NR integrable system.

The six cases above can be divided into two classes. The first one contains the first and last possibilities, and the other one - the remaining ones. The cases belonging to the first class are related to each other by the exchange of ϕ_1 and ϕ_2 . This corresponds to exchanging the two S^2 inside \mathbb{CP}^3 . Since these spheres enter symmetrically, the two cases are equivalent. In terms of (φ, ϕ_3) , the four cases from the second class are actually identical. That is why, all of them can be described simultaneously by choosing one representative from the class.

Let us first give the M2-brane solutions for cases in the first class. Since they correspond to our example in the previous section, the membrane configuration reads

$$\begin{aligned} W_1 &= Rr_1(\xi) \exp \{i\varphi_1(\tau, \xi)\} = R\sqrt{1 - r_0^2} \sin \theta(\xi) \exp \left\{ i \left[\frac{2}{k}\varphi(\tau, \xi) + \frac{\phi(\tau, \xi)}{2} \right] \right\}, \\ W_2 &= Rr_2(\xi) \exp \{i\varphi_2(\tau, \xi)\} = R\sqrt{1 - r_0^2} \cos \theta(\xi) \exp \left\{ i \left[\frac{2}{k}\varphi(\tau, \xi) - \frac{\phi(\tau, \xi)}{2} \right] \right\}, \\ W_3 &= Rr_0 \sin(\gamma\sigma_2), \quad W_4 = Rr_0 \cos(\gamma\sigma_2), \end{aligned}$$

where ϕ is equal to ϕ_1 or ϕ_2 .

From the NR system viewpoint, the membrane solutions for the second class configurations differ from the ones just given by the exchange of W_2, W_3 , and by the replacement $\phi/2 \rightarrow \phi_3$. In other words, we have

$$\begin{aligned} W_1 &= Rr_1(\xi) \exp \{i\varphi_1(\tau, \xi)\} = R\sqrt{1 - r_0^2} \sin \theta(\xi) \exp \left\{ i \left[\frac{2}{k}\varphi(\tau, \xi) + \phi_3(\tau, \xi) \right] \right\}, \\ W_2 &= Rr_0 \sin(\gamma\sigma_2), \\ W_3 &= Rr_3(\xi) \exp \{i\varphi_3(\tau, \xi)\} = R\sqrt{1 - r_0^2} \cos \theta(\xi) \exp \left\{ i \left[\frac{2}{k}\varphi(\tau, \xi) - \phi_3(\tau, \xi) \right] \right\}, \\ W_4 &= Rr_0 \cos(\gamma\sigma_2), \end{aligned}$$

The explicit solutions for $\theta(\xi)$ and $\varphi_{1,2,3}(\tau, \xi)$, of the M2-brane GM and SS, along with the energy-charge relations for the infinite and finite sizes are given in the appendix. Here, we will present them in terms of φ and $\phi_{1,2,3}$.

In accordance with (A.1), we have for the M2-brane GM with two angular momenta the following dispersion relation

$$\sqrt{1 - r_0^2}E - \frac{1}{2} \left(\frac{2}{k}J_\varphi + J_\phi \right) = \sqrt{\frac{1}{4} \left(\frac{2}{k}J_\varphi - J_\phi \right)^2 + 8\lambda k^2 [r_0(1 - r_0^2)\gamma]^2 \sin^2 \frac{p}{2}}, \quad (4.2)$$

where J_ϕ can be equal to $J_{\phi_1}/2, J_{\phi_2}/2$ or J_{ϕ_3} . In writing (4.2), we have used that

$$R = l_p (2^5 \pi^2 k N)^{1/6}, \quad T_2 = \frac{1}{(2\pi)^2 l_p^3},$$

and the 't Hooft coupling is defined by $\lambda = N/k$.

If we introduce the notations

$$\mathcal{E} = a\sqrt{1 - r_0^2}E, \quad \mathcal{J}_\varphi = a\frac{J_\varphi}{2}, \quad \mathcal{J}_\phi = a\frac{J_\phi}{2}, \quad a = \frac{1}{\sqrt{2\lambda k r_0(1 - r_0^2)\gamma}}, \quad (4.3)$$

the above energy-charge relation takes the form

$$\mathcal{E} - \mathcal{J}_1(k) = \sqrt{\mathcal{J}_2^2(k) + 4 \sin^2 \frac{p}{2}},$$

where

$$\mathcal{J}_1(k) = \frac{2}{k} \mathcal{J}_\varphi + \mathcal{J}_\phi, \quad \mathcal{J}_2(k) = \frac{2}{k} \mathcal{J}_\varphi - \mathcal{J}_\phi. \quad (4.4)$$

By using (4.3), (4.4) and (A.9), we can write down the dispersion relation for the dyonic GM, including the leading finite-size correction as

$$\begin{aligned} \mathcal{E} - \mathcal{J}_1(k) &= \sqrt{\mathcal{J}_2^2(k) + 4 \sin^2 \frac{p}{2}} - \frac{16 \sin^4 \frac{p}{2}}{\sqrt{\mathcal{J}_2^2(k) + 4 \sin^2 \frac{p}{2}}} \\ &\exp \left[- \frac{2 \sin^2 \frac{p}{2} \left(\mathcal{J}_1(k) + \sqrt{\mathcal{J}_2^2(k) + 4 \sin^2 \frac{p}{2}} \right) \sqrt{\mathcal{J}_2^2(k) + 4 \sin^2 \frac{p}{2}}}{\mathcal{J}_2^2(k) + 4 \sin^4 \frac{p}{2}} \right]. \end{aligned}$$

The reason to introduce \mathcal{E} , \mathcal{J}_φ and \mathcal{J}_ϕ namely in this way is the following. For GM on the $R_t \times S^3$ subspace of $AdS_5 \times S^5$, in terms of

$$\mathcal{E} = \frac{2\pi}{\sqrt{\lambda}} E, \quad \mathcal{J}_1 = \frac{2\pi}{\sqrt{\lambda}} J_1, \quad \mathcal{J}_2 = \frac{2\pi}{\sqrt{\lambda}} J_2,$$

we have

$$\mathcal{E} - \mathcal{J}_1 = \sqrt{\mathcal{J}_2^2 + 4 \sin^2 \frac{p}{2}}.$$

The same result can be obtained for the GM on the $R_t \times \mathbb{CP}^3$ subspace of $AdS_4 \times \mathbb{CP}^3$, if we use the identification [29]

$$\mathcal{E} = \frac{E}{\sqrt{2\lambda}}, \quad \mathcal{J}_1 = \frac{J_1}{\sqrt{2\lambda}}, \quad \mathcal{J}_2 = \frac{J_2}{\sqrt{2\lambda}}.$$

In the all three cases, the second term under the square root is the same. In this description it is universal - for different backgrounds and for different extended objects.

Analogously, for the SS case one can find (see (A.11))

$$\begin{aligned} \mathcal{E} - \Delta\varphi_1 &= p + 8 \sin^2 \frac{p}{2} \tan \frac{p}{2} \exp \left(- \frac{(\Delta\varphi_1 + p) \tan \frac{p}{2}}{\mathcal{J}_2^2(k) \csc^2 p + \tan^2 \frac{p}{2}} \right) \\ \mathcal{J}_1(k) &= \sqrt{\mathcal{J}_2^2(k) + 4 \sin^2 \frac{p}{2}}. \end{aligned}$$

Let us point out that for $k = 1$ the above dispersion relations coincide with the ones obtained earlier in [21]. We can also reproduce the energy-charge relations for dyonic GM and SS strings on $R_t \times \mathbb{CP}^3$ by taking an appropriate limit. To show this, let us consider the second case in (4.1), for which

$$J_{\phi_1} = \frac{2}{k}J_\varphi + J_{\phi_3}, \quad J_{\phi_2} = \frac{2}{k}J_\varphi - J_{\phi_3}.$$

In accordance with our membrane embedding, the following identification should be made

$$J_1^{str} = \frac{J_{\phi_1}}{2}, \quad J_2^{str} = \frac{J_{\phi_2}}{2}.$$

Then in the limit $k \rightarrow \infty$, $r_0 \rightarrow 0$, such that $kr_0\gamma = 1$, we obtain from (4.2)

$$E - J_1^{str} = \sqrt{(J_2^{str})^2 + 8\lambda \sin^2 \frac{p}{2}}.$$

This is exactly what we have derived in [29] for dyonic GM strings on $R_t \times \mathbb{CP}^3$. Obviously, this also applies for the leading finite-size correction. In the same way, the SS string dispersion relation for $R_t \times \mathbb{CP}^3$ background can be reproduced.

5 Concluding Remarks

We presented here an M2-brane perspective on ABJM super Chern-Simons-matter theory, which for large N and finite level k is dual to M theory on $AdS_4 \times S^7/Z_k$. In particular, we found membrane configurations, which can be described by the same NR integrable model for any positive integer k . Based on this, we gave the explicit solutions and the dispersion relations, including finite-size corrections, for states with two angular momenta, exhibiting GM and SS properties.

It would be interesting to see if the above results could be generalized to M2-branes with three and four angular momenta. Also, one can try to include the conserved spin S , arising from the nontrivial dynamics on the AdS_4 part of the background. In this case, one must take into account the membrane interaction with the three-form gauge field.

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A M2-brane GM and SS

For the GM-like case by using that $\tilde{C}_2 = 0$, $\tilde{\kappa}^2 = \omega_1^2$ in (3.8), (3.9), one finds

$$\begin{aligned} \cos \theta(\xi) &= \frac{\cos \tilde{\theta}_0}{\cosh(D_0 \xi)}, \quad \sin^2 \tilde{\theta}_0 = \frac{\beta^2 \omega_1^2}{\tilde{A}^2(\omega_1^2 - \omega_2^2)}, \quad D_0 = \frac{\tilde{A} \sqrt{\omega_1^2 - \omega_2^2}}{\tilde{A}^2 - \beta^2} \cos \tilde{\theta}_0, \\ \varphi_1(\tau, \xi) &= \omega_1 \tau + \arctan \left[\cot \tilde{\theta}_0 \tanh(D_0 \xi) \right], \quad \varphi_2(\tau, \xi) = \omega_2 \left(\tau + \frac{\beta}{\tilde{A}^2 - \beta^2} \xi \right). \end{aligned}$$

For the SS-like solutions when $\tilde{C}_2 = 0$, $\tilde{\kappa}^2 = \omega_1^2 \tilde{A}^2 / \beta^2$, by solving the equations (3.8), (3.9), one arrives at

$$\begin{aligned} \cos \theta(\xi) &= \frac{\cos \tilde{\theta}_1}{\cosh(D_1 \xi)}, \quad \sin^2 \tilde{\theta}_1 = \frac{\tilde{A}^2 \omega_1^2}{\beta^2(\omega_1^2 - \omega_2^2)}, \quad D_1 = \frac{\tilde{A} \sqrt{\omega_1^2 - \omega_2^2}}{\tilde{A}^2 - \beta^2} \cos \tilde{\theta}_1, \\ \varphi_1(\tau, \xi) &= \omega_1 \left(\tau - \frac{\xi}{\beta} \right) - \arctan \left[\cot \tilde{\theta}_1 \tanh(D_1 \xi) \right], \quad \varphi_2(\tau, \xi) = \omega_2 \left(\tau + \frac{\beta}{\tilde{A}^2 - \beta^2} \xi \right). \end{aligned}$$

The energy-charge relations computed on the above membrane solutions were found in [50], and in our notations read

$$\sqrt{1 - r_0^2} E - \frac{J_1}{2} = \sqrt{\left(\frac{J_2}{2} \right)^2 + \frac{\tilde{\lambda}}{\pi^2} \sin^2 \frac{p}{2}}, \quad \frac{p}{2} = \frac{\pi}{2} - \tilde{\theta}_0, \quad (\text{A.1})$$

for the GM-like case, and

$$\sqrt{1 - r_0^2} E - \frac{\sqrt{\tilde{\lambda}}}{2\pi} \Delta \varphi_1 = \frac{\sqrt{\tilde{\lambda}} p}{\pi} \frac{p}{2}, \quad \frac{J_1}{2} = \sqrt{\left(\frac{J_2}{2} \right)^2 + \frac{\tilde{\lambda}}{\pi^2} \sin^2 \frac{p}{2}}, \quad \frac{p}{2} = \frac{\pi}{2} - \tilde{\theta}_1, \quad (\text{A.2})$$

for the SS-like solution, where

$$\tilde{\lambda} = [2\pi^2 T_2 R^3 r_0 (1 - r_0^2) \gamma]^2. \quad (\text{A.3})$$

A.1 Finite-Size Effects

For $\tilde{C}_2 = 0$, Eq.(3.8) can be written as

$$(\cos \theta)' = \mp \frac{\tilde{A} \sqrt{\omega_1^2 - \omega_2^2}}{\tilde{A}^2 - \beta^2} \sqrt{(z_+^2 - \cos^2 \theta)(\cos^2 \theta - z_-^2)}, \quad (\text{A.4})$$

where

$$z_{\pm}^2 = \frac{1}{2(1 - \frac{\omega_2^2}{\omega_1^2})} \left\{ q_1 + q_2 - \frac{\omega_2^2}{\omega_1^2} \pm \sqrt{(q_1 - q_2)^2 - \left[2(q_1 + q_2 - 2q_1q_2) - \frac{\omega_2^2}{\omega_1^2} \right] \frac{\omega_2^2}{\omega_1^2}} \right\},$$

$$q_1 = 1 - \tilde{\kappa}^2/\omega_1^2, \quad q_2 = 1 - \beta^2 \tilde{\kappa}^2/\tilde{A}^2 \omega_1^2.$$

The solution of (A.4) is

$$\cos \theta = z_+ dn(C\xi|m), \quad C = \mp \frac{\tilde{A}\sqrt{\omega_1^2 - \omega_2^2}}{\tilde{A}^2 - \beta^2} z_+, \quad m \equiv 1 - z_-^2/z_+^2. \quad (\text{A.5})$$

The solutions of Eqs.(3.9) now read

$$\mu_1 = \frac{2\beta/\tilde{A}}{z_+ \sqrt{1 - \omega_2^2/\omega_1^2}} \left[C\xi - \frac{\tilde{\kappa}^2/\omega_1^2}{1 - z_+^2} \Pi \left(am(C\xi), -\frac{z_+^2 - z_-^2}{1 - z_+^2} \middle| m \right) \right],$$

$$\mu_2 = \frac{2\beta\omega_2/\tilde{A}\omega_1}{z_+ \sqrt{1 - \omega_2^2/\omega_1^2}} C\xi,$$

where $\Pi(k, n|m)$ is the elliptic integral of third kind.

Our next task is to find out what kind of energy-charge relations can appear for the M2-brane solution in the limit when the energy $E \rightarrow \infty$. It turns out that the semiclassical behavior depends crucially on the sign of the difference $\tilde{A}^2 - \beta^2$.

A.1.1 The M2-brane GM

We begin with the M2-brane GM, i.e. $\tilde{A}^2 > \beta^2$. In this case, one obtains from (3.7) the following expressions for the conserved energy E and the angular momenta J_1, J_2

$$\mathcal{E} = \frac{2\tilde{\kappa}(1 - \beta^2/\tilde{A}^2)}{\omega_1 z_+ \sqrt{1 - \omega_2^2/\omega_1^2}} \mathbf{K} \left(1 - z_-^2/z_+^2 \right),$$

$$\mathcal{J}_1 = \frac{2z_+}{\sqrt{1 - \omega_2^2/\omega_1^2}} \left[\frac{1 - \beta^2 \tilde{\kappa}^2/\tilde{A}^2 \omega_1^2}{z_+^2} \mathbf{K} \left(1 - z_-^2/z_+^2 \right) - \mathbf{E} \left(1 - z_-^2/z_+^2 \right) \right], \quad (\text{A.6})$$

$$\mathcal{J}_2 = \frac{2z_+\omega_2/\omega_1}{\sqrt{1 - \omega_2^2/\omega_1^2}} \mathbf{E} \left(1 - z_-^2/z_+^2 \right).$$

Here, we have used the notations

$$\mathcal{E} = \frac{2\pi}{\sqrt{\tilde{\lambda}}} \sqrt{1 - r_0^2} E, \quad \mathcal{J}_1 = \frac{2\pi}{\sqrt{\tilde{\lambda}}} \frac{J_1}{2}, \quad \mathcal{J}_2 = \frac{2\pi}{\sqrt{\tilde{\lambda}}} \frac{J_2}{2}, \quad (\text{A.7})$$

where $\tilde{\lambda}$ is defined in (A.3). The computation of $\Delta\varphi_1$ gives

$$p \equiv \Delta\varphi_1 = 2 \int_{\theta_{min}}^{\theta_{max}} \frac{d\theta}{\theta'} \mu'_1 = \tag{A.8}$$

$$- \frac{2\beta/\tilde{A}}{z_+ \sqrt{1 - \omega_2^2/\omega_1^2}} \left[\frac{\tilde{\kappa}^2/\omega_1^2}{1 - z_+^2} \Pi \left(-\frac{z_+^2 - z_-^2}{1 - z_+^2} \middle| 1 - z_-^2/z_+^2 \right) - \mathbf{K} \left(1 - z_-^2/z_+^2 \right) \right].$$

In the above expressions, $\mathbf{K}(m)$, $\mathbf{E}(m)$ and $\Pi(n|m)$ are the complete elliptic integrals.

Expanding the elliptic integrals about $z_-^2 = 0$, one arrives at

$$\mathcal{E} - \mathcal{J}_1 = \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)} - \frac{16 \sin^4(p/2)}{\sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)}} \tag{A.9}$$

$$\exp \left[-\frac{2 \left(\mathcal{J}_1 + \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)} \right) \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)} \sin^2(p/2)}{\mathcal{J}_2^2 + 4 \sin^4(p/2)} \right].$$

It is easy to check that the energy-charge relation (A.9) coincides with the one found in [51], describing the finite-size effects for dyonic GM. The difference is that in the string case the relations between \mathcal{E} , \mathcal{J}_1 , \mathcal{J}_2 and E , J_1 , J_2 are given by

$$\mathcal{E} = \frac{2\pi}{\sqrt{\lambda}} E, \quad \mathcal{J}_1 = \frac{2\pi}{\sqrt{\lambda}} J_1, \quad \mathcal{J}_2 = \frac{2\pi}{\sqrt{\lambda}} J_2,$$

while for the M2-brane they are written in (A.7).

A.1.2 The M2-brane SS

Let us turn our attention to the M2-brane SS, when $\tilde{A}^2 < \beta^2$. The computation of the conserved quantities (3.7) and $\Delta\varphi_1$ now gives

$$\mathcal{E} = \frac{2\tilde{\kappa}(\beta^2/\tilde{A}^2 - 1)}{\omega_1 \sqrt{1 - \omega_2^2/\omega_1^2} z_+} \mathbf{K} \left(1 - z_-^2/z_+^2 \right),$$

$$\mathcal{J}_1 = \frac{2z_+}{\sqrt{1 - \omega_2^2/\omega_1^2}} \left[\mathbf{E} \left(1 - z_-^2/z_+^2 \right) - \frac{1 - \beta^2 \tilde{\kappa}^2/\tilde{A}^2 \omega_1^2}{z_+^2} \mathbf{K} \left(1 - z_-^2/z_+^2 \right) \right],$$

$$\mathcal{J}_2 = -\frac{2z_+ \omega_2/\omega_1}{\sqrt{1 - \omega_2^2/\omega_1^2}} \mathbf{E} \left(1 - z_-^2/z_+^2 \right),$$

$$\Delta\varphi_1 = -\frac{2\beta/\tilde{A}}{\sqrt{1 - \omega_2^2/\omega_1^2} z_+} \left[\frac{\tilde{\kappa}^2/\omega_1^2}{1 - z_+^2} \Pi \left(-\frac{z_+^2 - z_-^2}{1 - z_+^2} \middle| 1 - z_-^2/z_+^2 \right) - \mathbf{K} \left(1 - z_-^2/z_+^2 \right) \right].$$

$\mathcal{E} - \Delta\varphi_1$ can be derived as

$$\begin{aligned} \mathcal{E} - \Delta\varphi_1 &= \arcsin N(\mathcal{J}_1, \mathcal{J}_2) + 2(\mathcal{J}_1^2 - \mathcal{J}_2^2) \sqrt{\frac{4}{[4 - (\mathcal{J}_1^2 - \mathcal{J}_2^2)]} - 1} \\ &\times \exp\left[-\frac{2(\mathcal{J}_1^2 - \mathcal{J}_2^2)N(\mathcal{J}_1, \mathcal{J}_2)}{(\mathcal{J}_1^2 - \mathcal{J}_2^2)^2 + 4\mathcal{J}_2^2} [\Delta\varphi + \arcsin N(\mathcal{J}_1, \mathcal{J}_2)]\right], \\ N(\mathcal{J}_1, \mathcal{J}_2) &\equiv \frac{1}{2} [4 - (\mathcal{J}_1^2 - \mathcal{J}_2^2)] \sqrt{\frac{4}{[4 - (\mathcal{J}_1^2 - \mathcal{J}_2^2)]} - 1}. \end{aligned} \quad (\text{A.10})$$

Finally, by using the SS relation between the angular momenta

$$\mathcal{J}_1 = \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)},$$

we obtain

$$\mathcal{E} - \Delta\varphi_1 = p + 8 \sin^2 \frac{p}{2} \tan \frac{p}{2} \exp\left(-\frac{\tan \frac{p}{2} (\Delta\varphi_1 + p)}{\tan^2 \frac{p}{2} + \mathcal{J}_2^2 \csc^2 p}\right). \quad (\text{A.11})$$

This result coincides with the string result found in [52]. As in the GM case, the difference is in the identification (A.7).

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