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An alternative S-matrix for $\mathcal{N} = 6$ Chern-Simons theory ?

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Abstract

We have recently proposed an S-matrix for the planar limit of the $\mathcal{N} = 6$ superconformal Chern-Simons theory of Aharony, Bergman, Jafferis and Maldacena which leads to the all-loop Bethe ansatz equations conjectured by Gromov and Vieira. An unusual feature of this proposal is that the scattering of A and B particles is reflectionless. We consider here an alternative S-matrix, for which $A-B$ scattering is not reflectionless. We argue that this S-matrix does not lead to the Bethe ansatz equations which are consistent with perturbative computations.

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1 Introduction

The fact that the 3-dimensional $\mathcal{N} = 6$ superconformal Chern-Simons (CS) theory of Aharony, Bergman, Jafferis and Maldacena [1] has a planar limit suggests that it may have further features in common with 4-dimensional $\mathcal{N} = 4$ superconformal Yang-Mills (YM) theory. Indeed, it was shown by Minahan and Zarembo [2] (see also [3]) that the two-loop anomalous dimensions of the scalar operators in planar $\mathcal{N} = 6$ CS theory are described by a certain integrable spin chain. Furthermore, they conjectured two-loop Bethe ansatz equations (BAEs) for the full theory. Gromov and Vieira [4] subsequently conjectured all-loop BAEs, which reduce to those of Minahan and Zarembo in the weak-coupling limit. Recently, three groups [5, 6, 7] computed the one-loop correction to the energy of a folded spinning string, and seemed to find disagreement with the prediction of the all-loop BAEs. This controversy may be resolved by a non-zero one-loop correction in the central interpolating function $h(\lambda)$ as suggested recently in [8]. (See also [9].)

Based on the spectrum and symmetries of the model [2, 10, 11, 12], we proposed an S-matrix [13] which reproduces the all-loop BAEs. That S-matrix has the unusual feature that the scattering of A and B particles is reflectionless,

$$A\,B \rightarrow B\,A$$

(instead of $A\,B \rightarrow B\,A + A\,B$). The purpose of this note is to search for other S-matrices which are consistent with the given symmetries and to derive the corresponding BAEs. In particular, we consider an alternative S-matrix for which $A-B$ scattering is not reflectionless. We derive the corresponding BAEs, and show that they do not reduce to those of Minahan and Zarembo [2] in the weak-coupling limit. This provides increased confidence in our earlier proposal for the S-matrix [13], and in the corresponding all-loop BAEs [4].

The outline of this paper is as follows. In Section 2 we present the new candidate S-matrix. In Section 3 we derive the corresponding all-loops BAEs by diagonalizing the Bethe-Yang matrix, and perform the weak-coupling limit. We conclude in Section 4 with a brief discussion of our results.

2 S-matrix

We represent the elementary excitations by Zamolodchikov-Faddeev operators $A^{\dagger}_{a\,i}(p)$, where $a \in \{1, 2\}$ is an $SU(2)$ flavor index ($a = 1$ corresponds to an A-particle, and $a = 2$ corresponds to a B-particle), and $i \in \{1, 2, 3, 4\}$ is the $SU(2|2)$ index. When acting on the vacuum state $|0\rangle$, these operators create corresponding asymptotic particle states of momentum p

and energy E given by [10, 11, 12, 14]

$$E = \sqrt{\frac{1}{4} + 4g^2 \sin^2 \frac{p}{2}}\,, \tag{2.1}$$

where g is a function of the 't Hooft coupling

$$g = h(\lambda)\,, \tag{2.2}$$

with $h(\lambda) \sim \lambda$ for small λ, and $h(\lambda) \sim \sqrt{\lambda/2}$ for large λ.

In order to have both $SU(2)$ and $SU(2|2)$ symmetries, the S-matrix should have the product form

$$A^\dagger_{a\,i}(p_1)\, A^\dagger_{b\,j}(p_2) = S_0(p_1, p_2)\, S_{a\,b}^{a'\,b'}(p_1, p_2)\, \widehat{S}_{i\,j}^{i'\,j'}(p_1, p_2)\, A^\dagger_{b'\,j'}(p_2)\, A^\dagger_{a'\,i'}(p_1)\,, \tag{2.3}$$

where $\widehat{S}_{i\,j}^{i'\,j'}(p_1, p_2)$ is the graded version of the $SU(2|2)$ S-matrix given in [15] (see also [16, 17, 18]) with g given by (2.2), $S_{a\,b}^{a'\,b'}(p_1, p_2)$ is an $SU(2)$ S-matrix which we shall discuss shortly, and $S_0(p_1, p_2)$ is an unknown scalar factor.

As is well-known (see, e.g., [19]), $SU(2)$ symmetry and factorizability almost completely fix the structure of $S_{a\,b}^{a'\,b'}(p_1, p_2)$. Indeed, $SU(2)$ symmetry implies that, up to an overall scalar factor,

$$S_{a\,b}^{a'\,b'}(p_1, p_2) = i\delta_a^{b'}\delta_b^{a'} + f(p_1, p_2)\delta_a^{a'}\delta_b^{b'}\,, \tag{2.4}$$

where $f(p_1, p_2)$ is an arbitrary scalar function of p_1, p_2. The Yang-Baxter equation

$$S_{12}(p_1, p_2)\, S_{13}(p_1, p_3)\, S_{23}(p_2, p_3) \; = S_{23}(p_2, p_3)\, S_{13}(p_1, p_3)\, S_{12}(p_1, p_2) \tag{2.5}$$

then implies that

$$f(p_1, p_2) = f(p_1, p_3) - f(p_2, p_3)\,, \tag{2.6}$$

which in turn implies that

$$f(p_1, p_2) = \alpha(p_1) - \alpha(p_2)\,, \tag{2.7}$$

where $\alpha(p)$ is an arbitrary function of p. In the weak-coupling limit, $\alpha(p)$ must be a linear function of p, say

$$\alpha(p) = p\,, \quad \text{(weak coupling)} \tag{2.8}$$

in order that $f(p_1, p_2)$ be a function of $p_1 - p_2$, i.e., that the S-matrix have the "difference" property. We conclude that

$$S_{a\,b}^{a'\,b'}(p_1, p_2) = i\delta_a^{b'}\delta_b^{a'} + (\alpha(p_1) - \alpha(p_2))\, \delta_a^{a'}\delta_b^{b'}\,. \tag{2.9}$$

In matrix form,

$$S(p_1,p_2) = \begin{pmatrix} a(p_1,p_2) & 0 & 0 & 0 \\ 0 & b(p_1,p_2) & i & 0 \\ 0 & i & b(p_1,p_2) & 0 \\ 0 & 0 & 0 & a(p_1,p_2) \end{pmatrix}, \tag{2.10}$$

where

$$a(p_1,p_2) = \alpha(p_1) - \alpha(p_2) + i\,, \qquad b(p_1,p_2) = \alpha(p_1) - \alpha(p_2)\,. \tag{2.11}$$

Note that

$$b(p_1,p_2) = -b(p_2,p_1)\,. \tag{2.12}$$

The S-matrix (2.3), unlike the one which we proposed in [13], does allow for reflection in $A-B$ scattering. Examples of integrable models with S-matrices of product form include [20]. To determine the function α, one may need to impose charge conjugation symmetry between A- and B-particles which leads to a crossing relation. We will not pursue this here since our conclusion does not depend on the explicit form of α.

3 Asymptotic Bethe ansatz

We now proceed to derive the corresponding all-loop BAEs. The analysis is similar to the one for $\mathcal{N}=4$ YM theory [17, 18]; and as in [13], we follow closely the latter reference. We consider a set of N particles with momenta p_i $(i=1,\ldots,N)$ which are widely separated on a ring of length L'. Quantization conditions for these momenta follow from imposing periodic boundary conditions on the wavefunction. Taking a particle with momentum p_k around the ring leads to the Bethe-Yang equations

$$e^{-ip_k L'} = \Lambda(\lambda = p_k, \{p_i\}; \{\lambda_j, \mu_j, \xi_j\})\,, \quad k=1,\ldots,N\,, \tag{3.1}$$

where $\Lambda(\lambda, \{p_i\}; \{\lambda_j, \mu_j, \xi_j\})$ are the eigenvalues of the transfer matrix

$$t(\lambda, \{p_i\}) = \Lambda_0(\lambda, \{p_i\})\, t_{SU(2)}(\lambda, \{p_i\}) \otimes t_{SU(2|2)}(\lambda, \{p_i\})\,, \tag{3.2}$$

where

$$\begin{aligned} \Lambda_0(\lambda, \{p_i\}) &= \prod_{i=1}^{N} S_0(\lambda, p_i)\,, \\ t_{SU(2)}(\lambda, \{p_i\}) &= \mathrm{tr}_a\, S_{a\,1}(\lambda, p_1)\cdots S_{a\,N}(\lambda, p_N)\,, \\ t_{SU(2|2)}(\lambda, \{p_i\}) &= \mathrm{str}_a\, \widehat{S}_{a\,1}(\lambda, p_1)\cdots \widehat{S}_{a\,N}(\lambda, p_N)\,. \end{aligned} \tag{3.3}$$

Hence, the eigenvalues are given by

$$\Lambda(\lambda, \{p_i\}; \{\lambda_j, \mu_j, \xi_j\}) = \Lambda_0(\lambda, \{p_i\})\, \Lambda_{SU(2)}(\lambda, \{p_i\}; \{\xi_j\})\, \Lambda_{SU(2|2)}(\lambda, \{p_i\}; \{\lambda_j, \mu_j\}) \,, \quad (3.4)$$

where the $SU(2)$ part is given by the well-known algebraic Bethe ansatz result

$$\Lambda_{SU(2)}(\lambda, \{p_i\}; \{\xi_j\}) = \prod_{i=1}^{N} a(\lambda, p_i) \prod_{j=1}^{m_0} s(\xi_j\,, \lambda) + \prod_{i=1}^{N} b(\lambda, p_i) \prod_{j=1}^{m_0} s(\lambda\,, \xi_j) \,, \quad (3.5)$$

with

$$s(p_1, p_2) = \frac{a(p_1, p_2)}{b(p_1, p_2)} = \frac{\alpha(p_1) - \alpha(p_2) + i}{\alpha(p_1) - \alpha(p_2)} \,, \quad (3.6)$$

and $\{\xi_j\}$ obey the BAEs

$$\prod_{i=1}^{N} s(\xi_k, p_i) = \prod_{\substack{j=1\\ j\neq k}}^{m_0} \frac{s(\xi_k, \xi_j)}{s(\xi_j, \xi_k)} \,, \qquad k = 1\,, \ldots\,, m_0 \,. \quad (3.7)$$

In particular, due to the property (2.12), the eigenvalues at $\lambda = p_k$ are given by

$$\Lambda_{SU(2)}(\lambda = p_k, \{p_i\}; \{\xi_j\}) = \prod_{i=1}^{N} a(p_k, p_i) \prod_{j=1}^{m_0} s(\xi_j\,, p_k) \,. \quad (3.8)$$

Moreover, the $SU(2|2)$ part is given by [18]

$$\begin{aligned}
\Lambda_{SU(2|2)}(\lambda, \{p_i\}; \{\lambda_j, \mu_j\}) &= \prod_{i=1}^{N} \left[\frac{x^+(\lambda) - x^-(p_i)}{x^-(\lambda) - x^+(p_i)} \frac{\eta(p_i)}{\eta(\lambda)}\right] \prod_{j=1}^{m_1} \left[\eta(\lambda) \frac{x^-(\lambda) - x^+(\lambda_j)}{x^+(\lambda) - x^+(\lambda_j)}\right] \\
&- \prod_{i=1}^{N} \left[\frac{x^+(\lambda) - x^+(p_i)}{x^-(\lambda) - x^+(p_i)} \frac{1}{\eta(\lambda)}\right] \left\{ \prod_{j=1}^{m_1} \left[\eta(\lambda) \frac{x^-(\lambda) - x^+(\lambda_j)}{x^+(\lambda) - x^+(\lambda_j)}\right] \prod_{l=1}^{m_2} \frac{x^+(\lambda) + \frac{1}{x^+(\lambda)} - \tilde{\mu}_l + \frac{i}{2g}}{x^+(\lambda) + \frac{1}{x^+(\lambda)} - \tilde{\mu}_l - \frac{i}{2g}} \right. \\
&+ \left. \prod_{j=1}^{m_1} \left[\eta(\lambda) \frac{x^+(\lambda_j) - \frac{1}{x^+(\lambda)}}{x^+(\lambda_j) - \frac{1}{x^-(\lambda)}}\right] \prod_{l=1}^{m_2} \frac{x^-(\lambda) + \frac{1}{x^-(\lambda)} - \tilde{\mu}_l - \frac{i}{2g}}{x^-(\lambda) + \frac{1}{x^-(\lambda)} - \tilde{\mu}_l + \frac{i}{2g}} \right\} \\
&+ \prod_{i=1}^{N} \left[\frac{x^+(\lambda) - x^+(p_i)}{x^-(\lambda) - x^+(p_i)} \frac{1 - \frac{1}{x^-(\lambda) x^+(p_i)}}{1 - \frac{1}{x^-(\lambda) x^-(p_i)}} \frac{\eta(p_i)}{\eta(\lambda)}\right] \prod_{j=1}^{m_1} \left[\eta(\lambda) \frac{x^+(\lambda_j) - \frac{1}{x^+(\lambda)}}{x^+(\lambda_j) - \frac{1}{x^-(\lambda)}}\right] \,, \quad (3.9)
\end{aligned}$$

where $\eta(\lambda) = e^{i\lambda/2}$, and the corresponding BAEs are given by

$$\begin{aligned}
e^{iP/2} \prod_{i=1}^{N} \frac{x^+(\lambda_j) - x^-(p_i)}{x^+(\lambda_j) - x^+(p_i)} &= \prod_{l=1}^{m_2} \frac{x^+(\lambda_j) + \frac{1}{x^+(\lambda_j)} - \tilde{\mu}_l + \frac{i}{2g}}{x^+(\lambda_j) + \frac{1}{x^+(\lambda_j)} - \tilde{\mu}_l - \frac{i}{2g}} \,, \qquad j = 1, \ldots, m_1 \,, \\
\prod_{j=1}^{m_1} \frac{\tilde{\mu}_l - x^+(\lambda_j) - \frac{1}{x^+(\lambda_j)} + \frac{i}{2g}}{\tilde{\mu}_l - x^+(\lambda_j) - \frac{1}{x^+(\lambda_j)} - \frac{i}{2g}} &= \prod_{\substack{k=1\\ k\neq l}}^{m_2} \frac{\tilde{\mu}_l - \tilde{\mu}_k + \frac{i}{g}}{\tilde{\mu}_l - \tilde{\mu}_k - \frac{i}{g}} \,, \qquad l = 1, \ldots, m_2 \,, \quad (3.10)
\end{aligned}$$

where

$$\frac{x^{+}(\lambda)}{x^{-}(\lambda)} = e^{i\lambda}\,, \quad x^{+}(\lambda) + \frac{1}{x^{+}(\lambda)} - x^{-}(\lambda) - \frac{1}{x^{-}(\lambda)} = \frac{i}{g}\,, \qquad P = \sum_{i=1}^{N} p_i\,. \tag{3.11}$$

In particular, the eigenvalue at $\lambda = p_k$ is given simply by

$$\Lambda_{SU(2|2)}(\lambda = p_k, \{p_i\}; \{\lambda_j, \mu_j\}) = \prod_{i=1}^{N} \left[\frac{x^{+}(p_k) - x^{-}(p_i)}{x^{-}(p_k) - x^{+}(p_i)} \frac{\eta(p_i)}{\eta(p_k)} \right] \prod_{j=1}^{m_1} \left[\eta(p_k) \frac{x^{-}(p_k) - x^{+}(\lambda_j)}{x^{+}(p_k) - x^{+}(\lambda_j)} \right] . \tag{3.12}$$

In view of (3.8), (3.12), the Bethe-Yang equations (3.1) take the form

$$\begin{aligned} e^{ip_k\left(-L' + \frac{N}{2} - \frac{m_1}{2}\right)} &= e^{iP/2} \prod_{i=1}^{N} \left\{ S_0(p_k, p_i)\, a(p_k, p_i) \left[\frac{x^{+}(p_k) - x^{-}(p_i)}{x^{-}(p_k) - x^{+}(p_i)} \right] \right\} \\ &\quad \times \prod_{j=1}^{m_0} s(\xi_j\,, p_k) \prod_{j=1}^{m_1} \frac{x^{-}(p_k) - x^{+}(\lambda_j)}{x^{+}(p_k) - x^{+}(\lambda_j)}\,, \qquad k = 1, \ldots, N\,, \end{aligned} \tag{3.13}$$

where $\{\lambda_j, \mu_j, \xi_j\}$ are determined by the BAEs (3.7), (3.10).

Following [18, 13], we make the identifications

$$\begin{aligned} x^{\pm}(p_k) &= x^{\pm}_{4,k}\,, \quad k = 1, \ldots, K_4 \equiv N\,, \\ x^{+}(\lambda_j) &= \frac{1}{x_{1,j}}\,, \quad j = 1, \ldots, K_1\,, \\ x^{+}(\lambda_{K_1+j}) &= x_{3,j}\,, \quad j = 1, \ldots, K_3\,, \quad K_1 + K_3 \equiv m_1\,, \\ \tilde{\mu}_j &= \frac{u_{2,j}}{g}\,, \quad j = 1, \ldots, K_2 \equiv m_2\,, \end{aligned} \tag{3.14}$$

and also define

$$u_{4,j} = x^{+}_{4,j} + \frac{1}{x^{+}_{4,j}} - \frac{i}{2} = x^{-}_{4,j} + \frac{1}{x^{-}_{4,j}} + \frac{i}{2}\,, \tag{3.15}$$

and $u_{i,j} = g\left(x_{i,j} + \frac{1}{x_{i,j}}\right)$ for $i = 1, 3$. We assume the zero-momentum condition

$$P = \sum_{j=1}^{K_4} p_{4,j} = 0\,, \tag{3.16}$$

and (for aesthetic reasons) we perform the shift

$$\alpha(\xi_j) \to \alpha(\xi_j) - \frac{i}{2}\,. \tag{3.17}$$

The Bethe-Yang equations (3.13) become

$$e^{ip_{4,k}\left(-L'+\frac{K_4+K_1-K_3}{2}\right)} = \prod_{i=1}^{K_4}\left\{S_0(p_{4,k},p_{4,i})\ [\alpha(p_{4,k})-\alpha(p_{4,i})+i]\left(\frac{x_{4,k}^+-x_{4,i}^-}{x_{4,k}^--x_{4,i}^+}\right)\right\}$$
$$\times\prod_{j=1}^{m_0}\frac{\alpha(\xi_j)-\alpha(p_{4,k})+\frac{i}{2}}{\alpha(\xi_j)-\alpha(p_{4,k})-\frac{i}{2}}\prod_{j=1}^{K_1}\frac{1-\frac{1}{x_{4,k}^- x_{1,j}}}{1-\frac{1}{x_{4,k}^+ x_{1,j}}}\prod_{j=1}^{K_3}\frac{x_{4,k}^- - x_{3,j}}{x_{4,k}^+ - x_{3,j}}\,, \quad k=1,\ldots,K_4\,, \tag{3.18}$$

and the BAEs (3.7), (3.10) become

$$\begin{aligned}
\prod_{i=1}^{K_4}\frac{\alpha(\xi_k)-\alpha(p_{4,i})+\frac{i}{2}}{\alpha(\xi_k)-\alpha(p_{4,i})-\frac{i}{2}} &= \prod_{\substack{j=1\\ j\neq k}}^{m_0}\frac{\alpha(\xi_k)-\alpha(\xi_j)+i}{\alpha(\xi_k)-\alpha(\xi_j)-i}\,, \qquad k=1\,,\ldots\,,m_0\,,\\
\prod_{i=1}^{K_4}\frac{1-\frac{1}{x_{1,j}x_{4,i}^-}}{1-\frac{1}{x_{1,j}x_{4,i}^+}} &= \prod_{l=1}^{K_2}\frac{u_{1,j}-u_{2,l}+\frac{i}{2}}{u_{1,j}-u_{2,l}-\frac{i}{2}}\,, \qquad j=1,\ldots,K_1\,,\\
\prod_{i=1}^{K_4}\frac{x_{3,j}-x_{4,i}^-}{x_{3,j}-x_{4,i}^+} &= \prod_{l=1}^{K_2}\frac{u_{3,j}-u_{2,l}+\frac{i}{2}}{u_{3,j}-u_{2,l}-\frac{i}{2}}\,, \qquad j=1,\ldots,K_3\,,\\
\prod_{j=1}^{K_1}\frac{u_{2,l}-u_{1,j}+\frac{i}{2}}{u_{2,l}-u_{1,j}-\frac{i}{2}}\prod_{j=1}^{K_3}\frac{u_{2,l}-u_{3,j}+\frac{i}{2}}{u_{2,l}-u_{3,j}-\frac{i}{2}} &= \prod_{\substack{k=1\\ k\neq l}}^{K_2}\frac{u_{2,l}-u_{2,k}+i}{u_{2,l}-u_{2,k}-i}\,, \qquad l=1,\ldots,K_2\,,
\end{aligned} \tag{3.19}$$

respectively. Eqs. (3.18), (3.19) constitute our result for the all-loop BAEs corresponding to the S-matrix (2.3), (2.9).

The weak-coupling limit corresponds to [4]

$$x\to\frac{u}{g}\,, \qquad x^{\pm}\to\frac{1}{g}\left(u\pm\frac{i}{2}\right)\,, \tag{3.20}$$

with $g\to 0$ and u finite. Recalling (2.8), we obtain

$$\begin{aligned}
\left(\frac{u_{4,k}+\frac{i}{2}}{u_{4,k}-\frac{i}{2}}\right)^L &= \prod_{i=1}^{K_4}\left\{S_0(p_{4,k},p_{4,i})\ (p_{4,k}-p_{4,i}+i)\left(\frac{u_{4,k}-u_{4,i}+i}{u_{4,k}-u_{4,i}-i}\right)\right\}\\
&\quad\times\prod_{j=1}^{m_0}\frac{\xi_j-p_{4,k}+\frac{i}{2}}{\xi_j-p_{4,k}-\frac{i}{2}}\prod_{j=1}^{K_3}\frac{u_{4,k}-u_{3,j}-\frac{i}{2}}{u_{4,k}-u_{3,j}+\frac{i}{2}}\,, \quad k=1,\ldots,K_4\,,\\
1 &= \prod_{\substack{j=1\\ j\neq k}}^{m_0}\frac{\xi_k-\xi_j+i}{\xi_k-\xi_j-i}\prod_{i=1}^{K_4}\frac{\xi_k-p_{4,i}-\frac{i}{2}}{\xi_k-p_{4,i}+\frac{i}{2}}\,, \qquad k=1\,,\ldots\,,m_0\,,
\end{aligned}$$

$$
\begin{aligned}
1 &= \prod_{l=1}^{K_2} \frac{u_{1,j} - u_{2,l} + \frac{i}{2}}{u_{1,j} - u_{2,l} - \frac{i}{2}} \,, \qquad j = 1, \ldots, K_1 \,, \\
1 &= \prod_{l=1}^{K_2} \frac{u_{3,j} - u_{2,l} + \frac{i}{2}}{u_{3,j} - u_{2,l} - \frac{i}{2}} \prod_{i=1}^{K_4} \frac{u_{3,j} - u_{4,i} - \frac{i}{2}}{u_{3,j} - u_{4,i} + \frac{i}{2}} \,, \qquad j = 1, \ldots, K_3 \,, \\
1 &= \prod_{\substack{k=1\\k\neq l}}^{K_2} \frac{u_{2,l} - u_{2,k} - i}{u_{2,l} - u_{2,k} + i} \prod_{j=1}^{K_1} \frac{u_{2,l} - u_{1,j} + \frac{i}{2}}{u_{2,l} - u_{1,j} - \frac{i}{2}} \prod_{j=1}^{K_3} \frac{u_{2,l} - u_{3,j} + \frac{i}{2}}{u_{2,l} - u_{3,j} - \frac{i}{2}} \,, \qquad l = 1, \ldots, K_2 \,,
\end{aligned} \tag{3.21}
$$

where we have defined

$$
L = -L' + \frac{K_4 + K_1 - K_3}{2} \,, \tag{3.22}
$$

and used

$$
e^{ip_{4,k}} = \frac{u_{4,k} + \frac{i}{2}}{u_{4,k} - \frac{i}{2}} \,. \tag{3.23}
$$

Evidently, regardless of the choice of scalar factor $S_0(p_1, p_2)$, the set of BAEs (3.21) does not completely match any of the equivalent sets of BAEs proposed by Minahan and Zarembo [2].

4 Discussion

We have considered an alternative S-matrix which is symmetric under $SU(2|2)$. In contrast with our original proposal [13], this S-matrix has the tensor product form (2.3); and it has not only an $SU(2|2)$ part, but also an $SU(2)$ part which allows for reflection in $A - B$ scattering. We did not completely specify the S-matrix, since we did not determine the scalar factor $S_0(p_1, p_2)$ in (2.3) and the function $\alpha(p)$ in (2.9). A priori an S-matrix of this form is plausible for a physical system with the given symmetry. We can conclude that this is not the correct S-matrix for $\mathcal{N} = 6$ CS only after checking that the corresponding all-loop BAEs do not lead to the perturbative BAEs [2]. This gives greater confidence in the original proposal [13]. [1]

It is widely believed that the new AdS/CFT duality between type IIA string on $AdS_4 \times CP^3$ and $\mathcal{N} = 6$ CS needs more study to clarify several features which are different from $\mathrm{AdS}_5/\mathrm{CFT}_4$. In this respect, it may be interesting to consider the strong coupling limit of the all-loop BAEs obtained here and investigate the corresponding (classical) string structure.

[1] Further support for the proposal [13] has recently been found in computations of finite-size corrections to the dispersion relation of giant magnons [21, 22].

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References

[1] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, "$\mathcal{N} = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals," [arXiv:0806.1218].

[2] J. A. Minahan and K. Zarembo, "The Bethe ansatz for superconformal Chern-Simons," *JHEP* **0809**, 040 (2008) [arXiv:0806.3951].

[3] D. Bak and S. J. Rey, "Integrable Spin Chain in Superconformal Chern-Simons Theory," [arXiv:0807.2063].

[4] N. Gromov and P. Vieira, "The all loop AdS4/CFT3 Bethe ansatz," [arXiv:0807.0777].

[5] T. McLoughlin and R. Roiban, "Spinning strings at one-loop in $AdS_4 \times P^3$," [arXiv:0807.3965].

[6] L. F. Alday, G. Arutyunov and D. Bykov, "Semiclassical Quantization of Spinning Strings in $AdS_4 \times CP^3$," [arXiv:0807.4400].

[7] C. Krishnan, "AdS4/CFT3 at One Loop," [arXiv:0807.4561].

[8] T. McLoughlin, R. Roiban and A. A. Tseytlin, "Quantum spinning strings in $AdS_4 \times CP^3$: testing the Bethe Ansatz proposal," [arXiv:0809.4038].

[9] N. Gromov and V. Mikhaylov, "Comment on the Scaling Function in $AdS4 \times CP3$," [arXiv:0807.4897].

[10] T. Nishioka and T. Takayanagi, "On Type IIA Penrose Limit and $\mathcal{N} = 6$ Chern-Simons Theories," *JHEP* **0808**, 001 (2008) [arXiv:0806.3391].

[11] D. Gaiotto, S. Giombi and X. Yin, "Spin Chains in $\mathcal{N} = 6$ Superconformal Chern-Simons-Matter Theory," [arXiv:0806.4589].

[12] G. Grignani, T. Harmark and M. Orselli, "The SU(2) x SU(2) sector in the string dual of $\mathcal{N} = 6$ superconformal Chern-Simons theory," [arXiv:0806.4959].

[13] C. Ahn and R. I. Nepomechie, "$\mathcal{N}=6$ super Chern-Simons theory S-matrix and all-loop Bethe ansatz equations," *JHEP* **0809**, 010 (2008) [arXiv:0807.1924].

[14] D. Berenstein and D. Trancanelli, "Three-dimensional $\mathcal{N}=6$ SCFT's and their membrane dynamics," [arXiv:0808.2503].

[15] G. Arutyunov, S. Frolov and M. Zamaklar, 'The Zamolodchikov-Faddeev algebra for $AdS_5 \times S^5$ superstring," *JHEP* **0704**, 002 (2007) [arXiv:hep-th/0612229].

[16] M. Staudacher, "The factorized S-matrix of CFT/AdS," *JHEP* **0505**, 054 (2005) [arXiv:hep-th/0412188].

[17] N. Beisert, "The $su(2|2)$ dynamic S-matrix," *Adv. Theor. Math. Phys.* **12**, 945 (2008) [arXiv:hep-th/0511082];
N. Beisert, "The Analytic Bethe Ansatz for a Chain with Centrally Extended $su(2|2)$ Symmetry," *J. Stat. Mech.* **0701**, P017 (2007) [arXiv:nlin/0610017].

[18] M.J. Martins and C.S. Melo, "The Bethe ansatz approach for factorizable centrally extended S-matrices," *Nucl. Phys.* **785**, 246 (2007) [arXiv:hep-th/0703086].

[19] N. Beisert, "Integrability in AdS/CFT," lecture at the workshop *Strong Fields, Integrability and Strings*, Newton Institute (2007).

[20] C. Ahn, D. Bernard and A. LeClair, "Fractional supersymmetries in perturbed coset CFTs and integrable soliton theory," *Nucl. Phys.* **B346**, 409 (1990);
N. Reshetikhin, "S-matrices in integrable models of isotropic magnetic chains. I," *J. Phys.* **A24**, 3299 (1991).

[21] D. Bombardelli and D. Fioravanti, "Finite-Size Corrections of the $\mathbb{CP}^3$ Giant Magnons: the Lüscher terms," [arXiv:0810.0704].

[22] T. Lukowski and O. O. Sax, "Finite size giant magnons in the $SU(2) \times SU(2)$ sector of $AdS_4 \times CP^3$," [arXiv:0810.1246].