3장. Oscillations

linear : $F(x) \propto x$

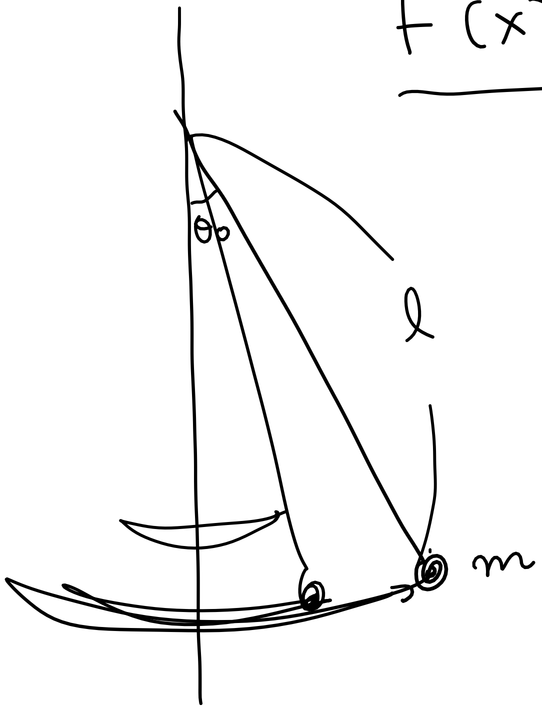
$$m\frac{d^2x}{dt^2} = F(x) = -kx$$

$\alpha > 0$ → unnatural

$\underset{=-k}{\alpha} < 0$ → natural

기함수

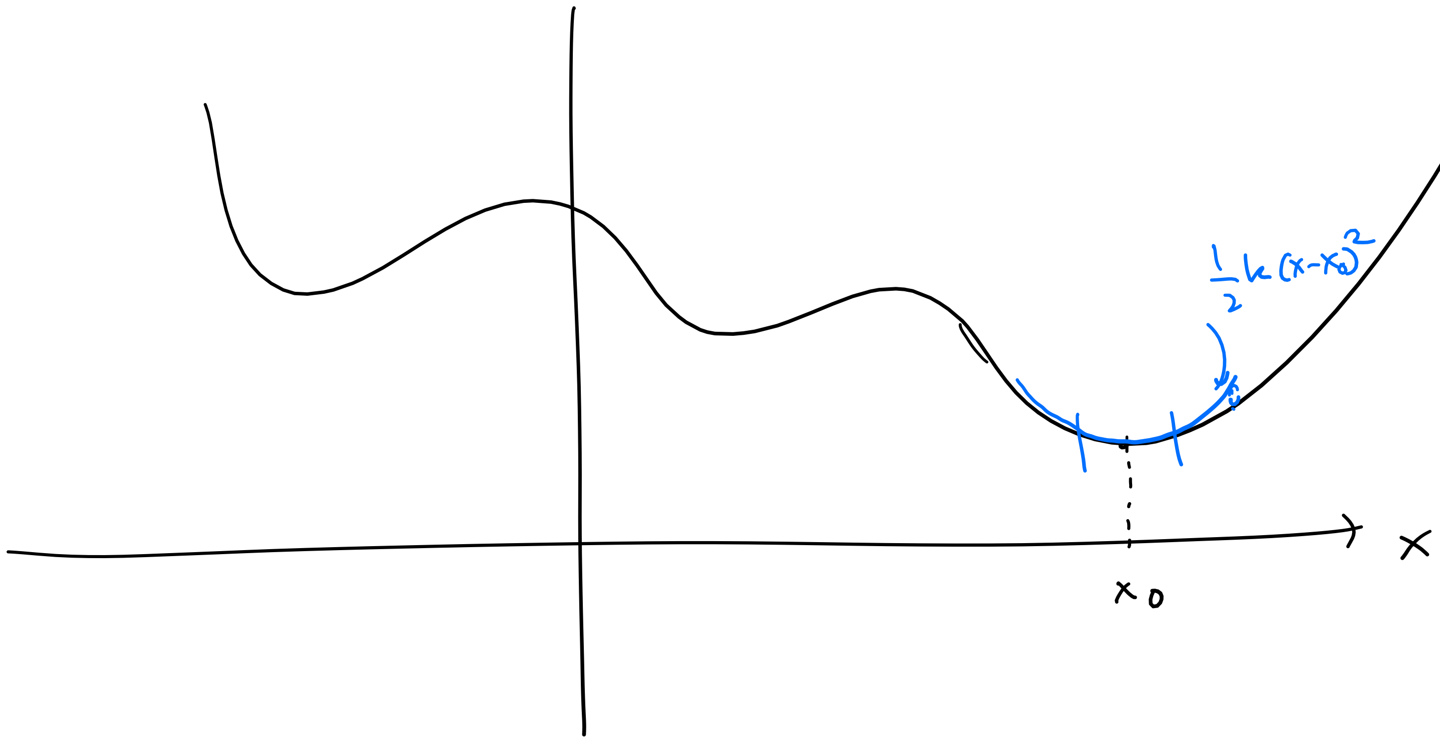
$$\underbrace{F(x) = -kx}\ \underbrace{-k_1x^3 - k_2x^5 + \cdots}$$



$T = 2\pi\sqrt{\frac{l}{g}}$ ← θ_0 에 무관

$$\sin\theta \approx \boxed{\theta}\ \cancel{- \frac{\theta^3}{6} + \cdots}$$

$$F(x) = -\frac{dV(x)}{dx} \longrightarrow V(x) = \frac{1}{2}kx^2$$

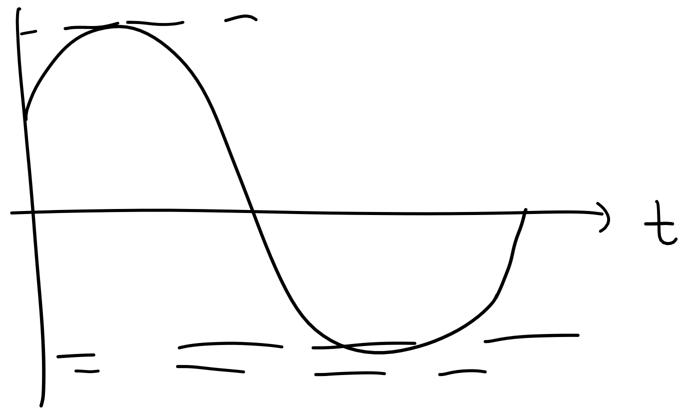


$$\omega_0 T_0 = 2\pi$$

$$T_0 = \frac{2\pi}{\omega_0}$$

$$\ddot{x} = -\frac{k}{m}x \longrightarrow \underline{x = A\sin(\omega_0 t + \phi_0)}$$

$$\sqrt{\frac{k}{m}} = \omega_0$$



MATLAB

```
t = 0:0.1:4*pi;
plot (t, 2*sin(1.0*t + pi/3))
```

초기조건

$$x(0) = x_0$$

$$\dot{x}(0) = v(0) = v_0$$

$$x = A\sin(\omega_0 t + \phi_0)$$

$$\dot{x} = \omega_0 A\cos(\omega_0 t + \phi_0)$$

$$x_0, v_0 \longleftrightarrow A, \phi_0$$

$$x_0 = A\sin\phi_0$$

$$\frac{v_0}{\omega_0} = A\cos\phi_0$$

$$x(0) = x_0 = A\sin(\phi_0)$$

$$\dot{x}(0) = v_0 = \omega_0 A\cos\phi_0$$

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_0}\right)^2}$$

$$\tan\phi_0 = \frac{x_0\omega_0}{v_0} \longrightarrow \phi_0 = \tan^{-1}\left(\frac{x_0\omega_0}{v_0}\right)$$

---

ode45을 이용한 2차 미방

$$\frac{dv}{dt} = f(v)$$

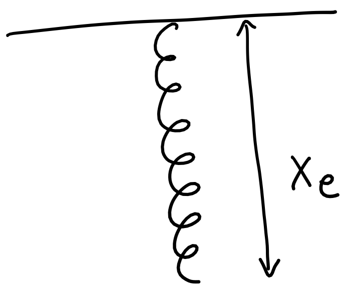
1원 2차 미방

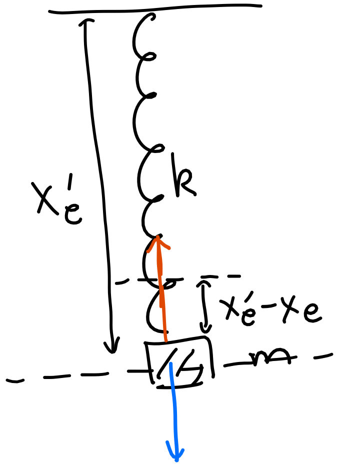
$$\frac{d^2x}{dt^2} = -\omega_0^2 x$$

2원 1차 미방

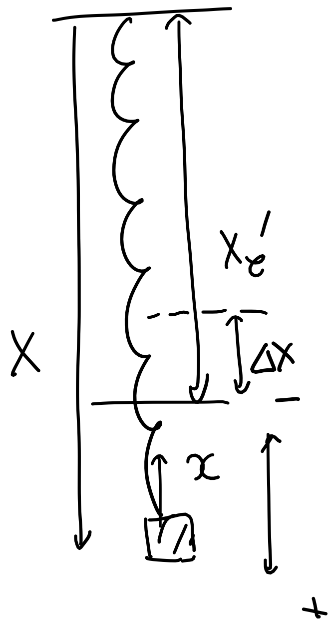
$$\boxed{\begin{aligned} \frac{dx}{dt} &= v \\ \frac{dv}{dt} &= -\omega_0^2 x \end{aligned}}$$

(x, v)





$$mg = k(\underbrace{x_e' - x_e}_{\Delta x})$$



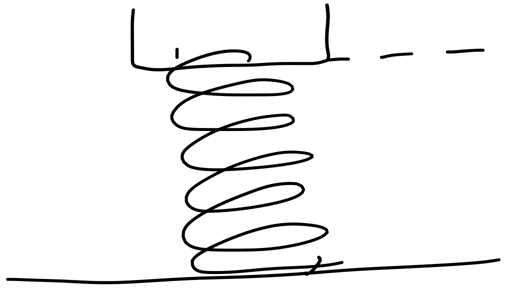
$$m\ddot{X} = -kx + \cancel{mg} - \cancel{k\Delta x}$$

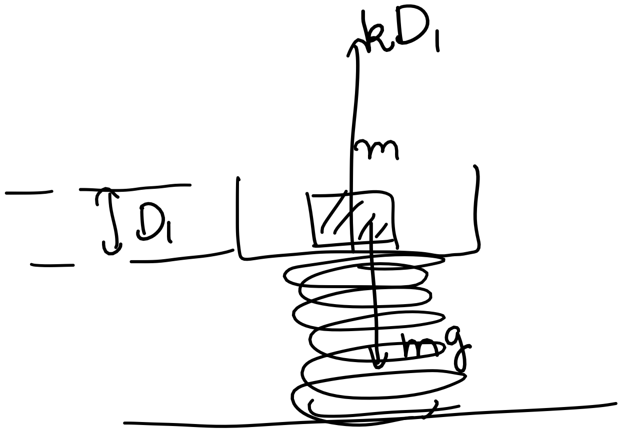
$$X = x + x_e'$$

$$\dot{X} = \dot{x} \qquad \ddot{X} = \ddot{x}$$

$$\Rightarrow \quad m\ddot{x} = -kx$$

Ex 3.2.1





$$kD_1 = mg$$

$$\rightarrow k = \frac{mg}{D_1}$$

$$\frac{k}{m} = \omega_0^2 = \frac{g}{D_1} \rightarrow \omega_0 = \sqrt{\frac{g}{D_1}}$$

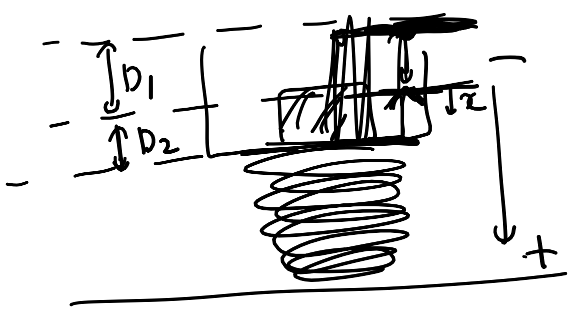
$$\downarrow$$

$$x = \underline{\underline{A}} \sin(\omega_0 t + \underline{\underline{\phi_0}})$$

$$x(0) = D_2 = A \sin\phi_0 = A$$

$$\dot{x}(0) = 0 = \omega_0 A \cos\phi_0$$

$$\Rightarrow \cos\phi_0 = 0 \rightarrow \phi_0 = \frac{\pi}{2} \quad -\frac{\pi}{2}$$



$$g \cdot \frac{D_2}{D_1}$$

$$\parallel$$

top 에서 $a = \omega_0^2 D_2$

$$\ddot{x} = -\omega_0^2 D_2 \underbrace{\cos(\omega_0 t)}_{-1}$$

한번더 미분

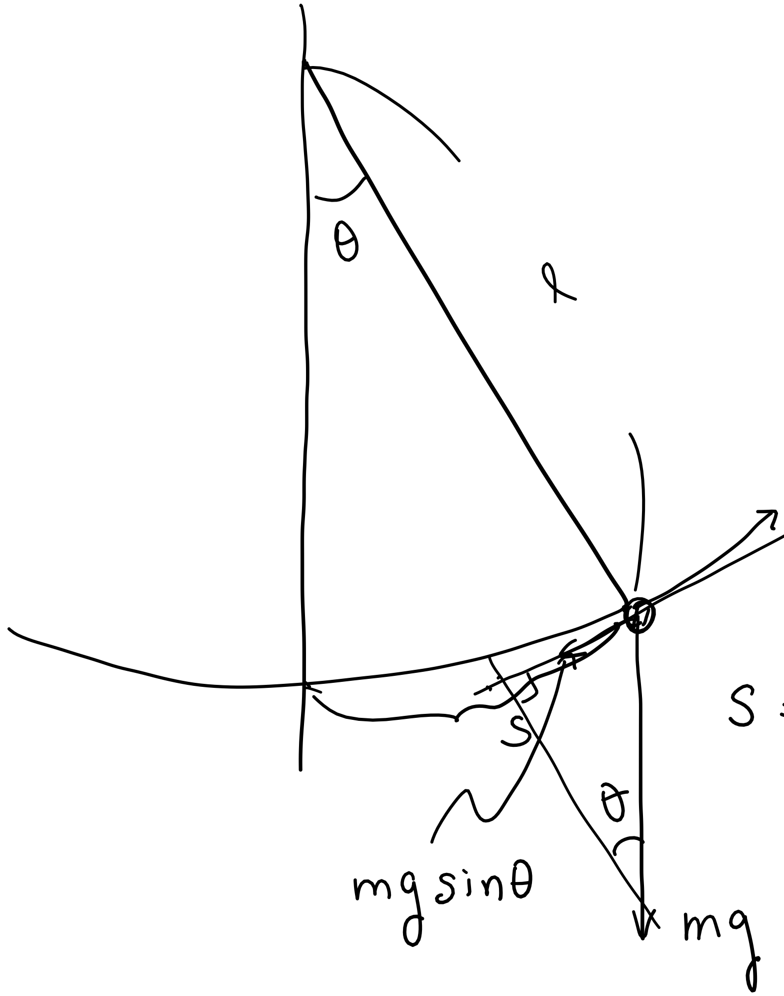
$$\dot{x} = -\omega_0 D_2 \sin(\omega_0 t) \leftarrow \therefore x = D_2 \sin\left(\omega_0 t + \frac{\pi}{2}\right) = D_2 \cos(\omega_0 t)$$

$$= -D_2 \sin\left(\omega_0 t - \frac{\pi}{2}\right)$$

$$x = 0 \rightarrow \omega_0 t = \frac{\pi}{2}$$

$$\dot{x}\left(t = \frac{\pi}{2\omega_0}\right) = -\omega_0 D_2 = -\sqrt{\frac{g}{D_1}} \cdot D_2$$

[Ex 3.2.2]



$S = l\theta$

$m\ddot{s} = -mg\sin\theta$

$\ddot{\theta} = -\frac{g}{l}\sin\theta = -\omega_0^2\sin\theta$

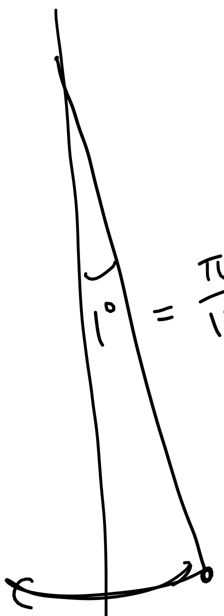
radian ($\pi = 180^\circ$)

if $\theta \ll 1$; $\sin\theta \approx \theta$

($A \ll 1$)

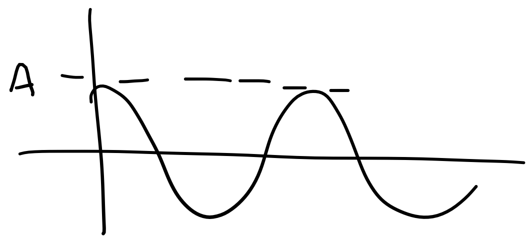
$\ddot{\theta} = -\omega_0^2\,\theta$

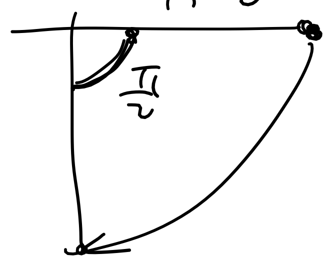
$\theta = A\sin(\omega_0 t + \phi_0)$



$1^\circ = \frac{\pi}{180} \ll 1$

$\sin\theta \cong \theta$





$\theta \cancel{=} + \frac{\pi}{2}\cos(\omega_0 t)$

$\sin\theta \neq \theta$

$$F=-kx \longrightarrow V=\underline{\frac{1}{2}kx^2}+\text{상수}$$

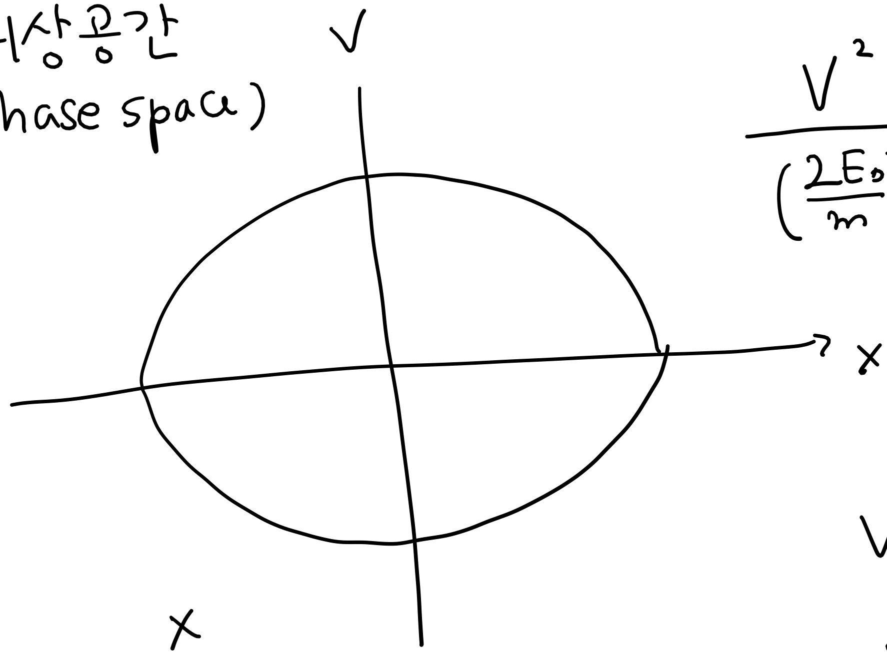
$$\text{상수}=0$$

$$E=K+V=\frac{1}{2}mv^2+\frac{1}{2}kx^2=\text{상수}\ (\text{에너지 보존})=E_0$$

$$\downarrow$$

$$\frac{v^2}{\left(\frac{2E_0}{m}\right)}+\frac{x^2}{\left(\frac{2E_0}{k}\right)}=1 \longleftrightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

위상공간 (phase space)



$$v=\sqrt{\frac{2E_0}{m}}\sqrt{1-\frac{x^2}{\left(\frac{2E_0}{k}\right)}}=\frac{dx}{dt}$$

$$\int_{x_0}^{x}\frac{dx}{\sqrt{1-\underbrace{\frac{x^2}{\left(\frac{2E_0}{k}\right)}}_{y^2}}}=\sqrt{\frac{2E_0}{m}}\int_0^t dt=\underbrace{\sqrt{\frac{2E_0}{m}}\cdot t}$$

$$=\sqrt{\frac{2E_0}{k}}\int_{y_0}^{y}\frac{dy}{\sqrt{1-y^2}}=\sqrt{\frac{2E_0}{k}}\int_{\phi_0}^{\theta}\frac{\cancel{\cos\theta}\,d\theta}{\cancel{\cos\theta}}=\underbrace{\sqrt{\frac{2E_0}{k}}(\theta-\phi_0)}$$

$$\underline{y=\sin\theta}$$

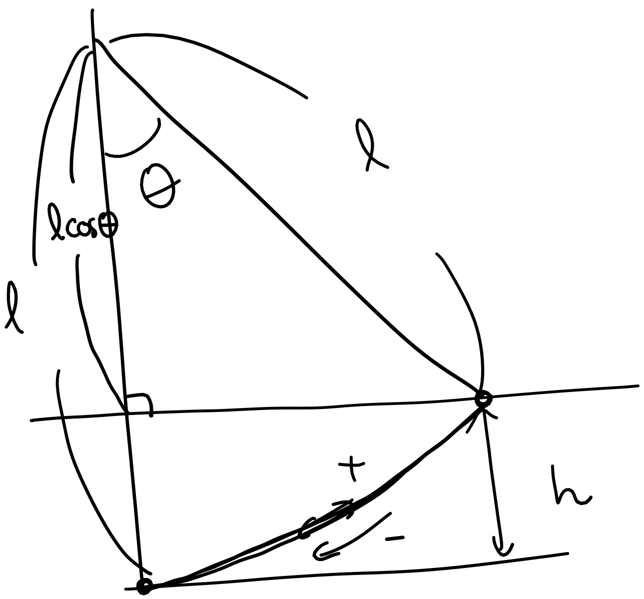
$$dy=\cos\theta\,d\theta$$

$$\frac{x}{\sqrt{\frac{2E_0}{k}}}=y \rightarrow dx=\sqrt{\frac{2E_0}{k}}\,dy$$

$$\therefore\ \theta=\omega_0 t+\phi_0 \rightarrow y=\sin(\omega_0 t+\phi_0)$$

$$\therefore\ x=A\sin(\omega_0 t+\phi_0), \quad \sqrt{\frac{2E_0}{k}}\equiv A$$

[Ex 3.3.1]



$$V = mgh = mg\ell(1-\cos\theta)$$

$$h = \ell - \ell\cos\theta$$

$$E = \frac{1}{2}m\dot{s}^2 + V = \frac{1}{2}m\ell^2\dot{\theta}^2 + V$$

$$\dot{\theta} = \pm\sqrt{\frac{2}{m\ell^2}(E-V)} \overset{+}{=} \sqrt{\frac{2E}{m\ell^2}\Big(1 - \frac{mg\ell}{E}(\underbrace{1-\cos\theta}_{2\sin^2\frac{\theta}{2}})\Big)} = \frac{d\theta}{dt}$$

$$= \sqrt{\underbrace{\frac{\cancel{2E}}{\cancel{m\ell^2}}\frac{2\cancel{mg\ell}}{\cancel{2E}}}_{\frac{2g}{\ell}}\Big(\underbrace{\frac{2E}{mg\ell}}_{\overset{\shortparallel}{c^2}} - \sin^2\frac{\theta}{2}\Big)}$$

$$\int_{\theta_0}^{\theta} \frac{d\theta}{\sqrt{c^2 - \sin^2\frac{\theta}{2}}} = \sqrt{\frac{2g}{\ell}}\cdot\int_0^t dt \longrightarrow \theta = \theta(t)$$

↳ elliptic integral.

↳ MATLAB.

주기운동 주기 T. $\sin(\underbrace{\omega_0 t}_{2\pi} + \phi_0)$

$\parallel$

$T = \frac{2\pi}{\omega_0}$

$t \to t+T$

$\omega_0 T = 2\pi$

평균값 :

$$\langle K \rangle = \frac{1}{T}\int_0^T K(t)\,dt = \frac{1}{T}\int_0^T \frac{1}{2}m\dot{x}^2\,dt$$

$x = A\sin(\omega_0 t + \phi_0)$

$\dot{x} = A\omega_0\cos(\omega_0 t + \phi_0)$

$$\langle K \rangle = \frac{1}{4}\cancel{m}A^2\omega_0^2 = \frac{1}{\cancel{T}}\cdot\frac{1}{2}mA^2\omega_0^2 \int_0^T \underbrace{\cos^2(\omega_0 t + \phi_0)}_{\frac{1}{2}(1+\cos(2\omega_0 t + 2\phi_0))}\,dt$$

$\omega_0^2 = \frac{k}{\cancel{m}}$

$\frac{1}{2}\cancel{T}$

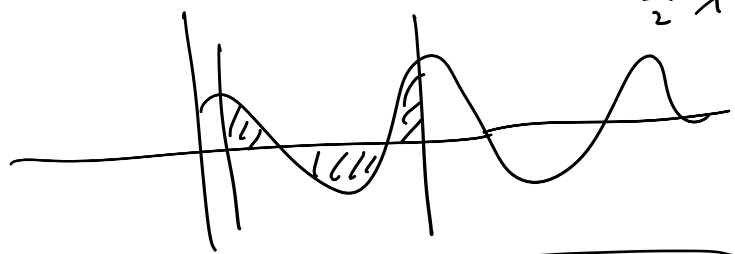
$\frac{1}{2\omega_0}\sin(2\omega_0 t + 2\phi_0)\Big|_0^T$

$= \frac{1}{2\omega_0}\left(\sin(4\pi + 2\phi_0) - \sin(2\phi_0)\right)$

$= 0$

$\parallel$

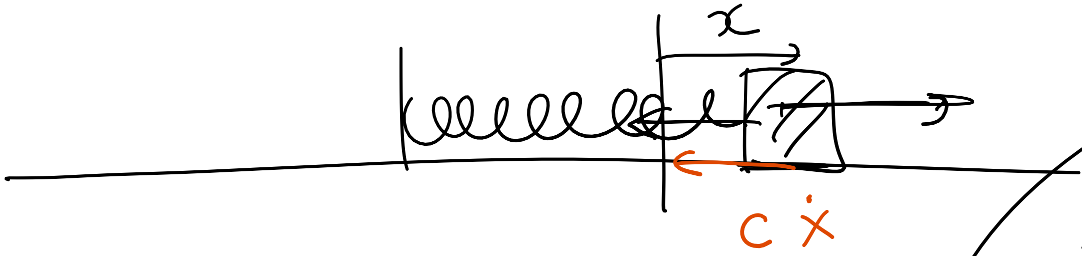
$$\langle V \rangle = \underbrace{\frac{1}{4}kA^2}$$



$$\langle V \rangle = \frac{1}{T}\int_0^T \frac{1}{2}k\underbrace{x^2}\,dt = \frac{1}{\cancel{T}}\cdot\frac{k}{2}\cdot A^2\,\frac{\cancel{T}}{2} = \underbrace{\frac{kA^2}{4}}$$

$A^2\sin^2(\omega_0 t + \phi_0) = A^2\frac{1}{2}(1 - \cancel{\cos(2\omega_0 t + 2\phi_0)})$

3.4. Damped Harmonic Motion



$$m\ddot{x} = -kx - c\dot{x}$$

$$\frac{c}{2m} \equiv \gamma \qquad \frac{k}{m} = \omega_0^2$$

$$\ddot{x} = -\omega_0^2 x - 2\gamma\dot{x}$$

$$\boxed{\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0}$$

linear

시간에 무관

$$x = Ae^{Dt}$$ 을 가정

$$\dot{x} = DAe^{Dt}$$

$$\ddot{x} = D^2 A e^{Dt}$$

$$\left[\underbrace{D^2 + 2\gamma D + \omega_0^2}_{=0}\right] Ae^{Dt} = 0$$

MATLAB.

$$\dot{x} = v \rightarrow \frac{dx}{dt} = v$$

$$\frac{dv}{dt} = -\omega_0^2 x - 2\gamma v$$

$$[x, v] \equiv z$$

$$\frac{dx}{dt} = z(2)$$

$$\frac{dv}{dt} = -\omega_0^2 z(1) - 2\gamma z(2)$$

$$D^2 + 2\gamma D + \omega_0^2 = 0$$

$$\parallel$$

$$\underbrace{D^2 + 2\gamma D + \gamma^2}_{(D+\gamma)^2} + \omega_0^2 - \gamma^2$$

$$(D+\gamma)^2 - (\underbrace{\gamma^2 - \omega_0^2}_{q^2}) = \left(D + \gamma - \sqrt{\gamma^2-\omega_0^2}\right)\left(D+\gamma+\sqrt{\gamma^2-\omega_0^2}\right)$$

① overdamped (과감쇠) $\gamma > \omega_0 \longrightarrow q = \text{실수} \Rightarrow \gamma > q$, $q = \sqrt{\gamma^2-\omega_0^2}$

② underdamped (덜감쇠) $\gamma < \omega_0 \longrightarrow q = \text{허수} = i\omega_d = i\sqrt{\omega_0^2-\gamma^2}$

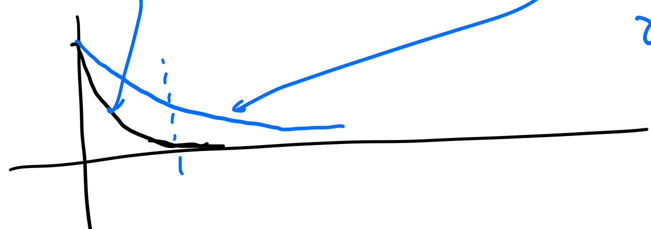
$\omega_d = \sqrt{\omega_0^2-\gamma^2}$

③ critical damped (딱감쇠) $\gamma = \omega_0 \longrightarrow q = 0$

---

① $D = -\gamma \pm q$ $\quad \therefore x = A e^{-(\gamma+q)t} + B e^{-(\gamma-q)t}$, $\quad \gamma - q \geq 0$

$\gamma + q > \gamma - q$



② $D = -\gamma \pm i\omega$

$$x = A e^{-(\gamma + i\omega_d)t} + B e^{-(\gamma - i\omega_d)t}$$

$$= e^{-\gamma t}\left[A e^{-i\omega_d t} + B e^{i\omega_d t}\right]$$

$$\sin\omega t = \frac{e^{i\omega t} - e^{-i\omega t}}{2i} \rightarrow e^{i\omega t} = \cos\omega t + i\sin\omega t$$

$$e^{-i\omega t} = \cos\omega t - i\sin\omega t$$

$$x = e^{-\gamma t}\left[A e^{-i\omega t} + B e^{i\omega t}\right] \qquad A = |A| e^{i\phi_1},\ B = |B| e^{i\phi_2}$$

$$x = \text{실수부}\left[e^{-\gamma t}\left(|A| e^{-i\omega t + i\phi_1} + |B| e^{i\omega t + i\phi_2}\right)\right]$$

$\cos\omega t\cos\phi_1 + \sin\omega t \sin\phi_1$ $\qquad$ $\cos\omega t \cos\phi_2 - \sin\omega t \sin\phi_2$

$$= e^{-\gamma t}\left(|A|\cos(\omega t - \phi_1) + |B|\cos(\omega t + \phi_2)\right)$$

$$A_1 \cos\omega t + B_1 \sin\omega t$$

$$= \underbrace{e^{-\gamma t} A}_{\text{진폭}} \sin(\omega_d t + \phi_0)$$

$$\sqrt{A_1^2+B_1^2}\left(\frac{A_1}{\sqrt{A_1^2+B_1^2}}\cos\omega t + \frac{B_1}{\sqrt{A_1^2+B_1^2}}\sin\omega t\right)$$

$$\frac{A_1}{\sqrt{A_1^2+B_1^2}} = \sin\phi_0, \quad \frac{B_1}{\sqrt{A_1^2+B_1^2}} = \cos\phi_0, \quad \sqrt{A_1^2+B_1^2} = A$$

수학 : 2차 미분방정식의 해는 2개 존재.

$$\ddot{x} = -\omega_0^2 x \longrightarrow x = A\cos\omega_0 t + B\sin\omega_0 t = A\cos(\omega_0 t + \phi_0)$$

딱 감쇠 ;
$\gamma = \omega_0$

$$x = Ae^{Dt} \rightarrow D = -\gamma \quad \text{(중근)}$$

$$= Ae^{-\gamma t}$$

$$\rightarrow x = Ate^{-\gamma t}$$ 도 해.

$$\dot{x} = A(1-\gamma t)e^{-\gamma t}$$

$$\ddot{x} = A(\underbrace{-\gamma - \gamma}_{-2\gamma} + \gamma^2 t)e^{-\gamma t}$$

$$\ddot{x} = \underbrace{-\gamma^2 x - 2\gamma \dot{x}}$$

$$= A(\gamma^2 t - 2\gamma)e^{-\gamma t}$$

$$= \left(-\gamma^2 At - 2\gamma A(1-\gamma t)\right)e^{-\gamma t}$$

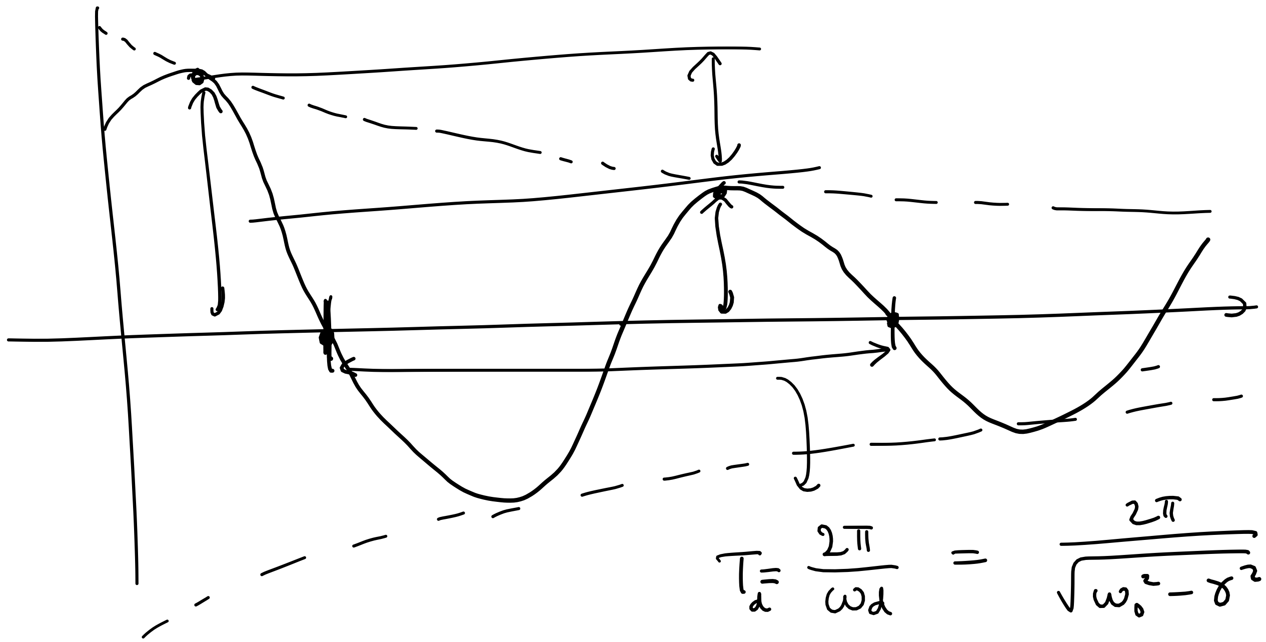
$$= (\gamma^2 At - 2\gamma A) = A(\gamma^2 t - 2\gamma)e^{-\gamma t} \checkmark$$

$$\underline{x = Ae^{-\gamma t} + Bte^{-\gamma t}}$$

$$x(0) = x_0 = A$$

$$\dot{x}(0) = v_0 = -\gamma A + B$$

덜감쇠 :



$$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{\omega_0^2-\gamma^2}} > \frac{2\pi}{\omega_0}$$

Energy : $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$

$$\frac{dE}{dt} = m\dot{x}\ddot{x} + kx\dot{x} = \dot{x}\left(\overbrace{m\ddot{x} + kx}^{-c\dot{x}}\right) = -c\dot{x}^2 < 0$$

$m\ddot{x} + kx + c\dot{x} = 0$ if $c = 0$

덜감쇠

$$x = Ae^{-\gamma t}\sin(\overbrace{\omega_d t + \phi_0}^{\theta})$$

$$\dot{x} = A(-\gamma\sin\theta + \omega_d\cos\theta)e^{-\gamma t}$$

$$\frac{dE}{dt} = -c\dot{x}^2 = -c\left[A\left(-\gamma\sin\theta + \omega_d\cos\theta\right)e^{-\gamma t}\right]^2$$

$\theta = \omega_d t + \phi_0$

$$\Delta E = \int_0^{T_d} \dot{E}\,dt = \int_{\phi_0}^{2\pi+\phi_0} \dot{E}\,\frac{d\theta}{\omega_d}$$

$d\theta = \omega_d\,dt$

$$= \frac{1}{\omega_d}\int_{\phi_0}^{2\pi+\phi_0} (-cA^2)\, \underset{\downarrow}{e^{-2\gamma t}} \left(-\gamma\sin\theta + \omega_d\cos\theta\right)^2 d\theta$$

$$e^{-2\gamma T_d} = e^{-2\gamma\frac{2\pi}{\omega_d}} \qquad 2\gamma\frac{2\pi}{\omega_d} \sim \frac{\gamma}{\omega_0} \ll 1$$

$\boxed{\gamma \ll \omega_0 \text{ 경우}}$

$$\omega_d = \sqrt{\omega_0^2 - \gamma^2} \approx \omega_0$$

$$\cong \frac{1}{\omega_d}(-cA^2)\,e^{-2\gamma t}\int_{\phi_0}^{2\pi+\phi_0}\left(-\gamma\sin\theta + \omega_d\cos\theta\right)^2 d\theta$$

$$= \gamma^2\underbrace{\sin^2\theta}_{\int d\theta\, =\, \pi} + \omega_d^2\underbrace{\cos^2\theta}_{=\,\pi} - 2\gamma\omega_d\,\sin\theta\cos\theta$$

$$= -\underbrace{\frac{2\pi}{\omega_d}}_{T_d}\,\frac{\overset{2m\gamma}{c}}{2}\,A^2 e^{-2\gamma t}\underbrace{\left(\omega_d^2+\gamma^2\right)}_{=\,\omega_0^2} = -\gamma m\omega_0^2 A^2 T_d\, e^{-2\gamma t}$$

(cf) $E = \frac{1}{2}m\omega_0^2\underbrace{A^2 e^{-2\gamma t}}_{A}$

$$\frac{\Delta E}{E} = -2\gamma T_d \qquad \left(2\gamma \equiv \frac{1}{\tau}\right)$$

$$= -\frac{T_d}{\tau}$$

Q가 크면 $\left(\frac{T_d}{\tau}\right.$ 가 작으면$\left.\right)$ → E가 잘 안죽는 운동

Q가 작으면 (〃 크면) → 〃 죽는 운동

$$Q \equiv \frac{2\pi}{\left(\frac{T_d}{\tau}\right)} \sim 10^4 \text{ for crystal}$$

$\frac{1}{e}$ 되려면 $10^4\, T_d$

[Ex. 3.4.1.] $\frac{2\pi}{\omega_0} = 1 \rightarrow \omega_0 = 2\pi = \gamma$

$$\underline{x = A e^{-\gamma t} + B t e^{-\gamma t}}$$

$x(0) = x_0 = A \rightarrow A = x_0$

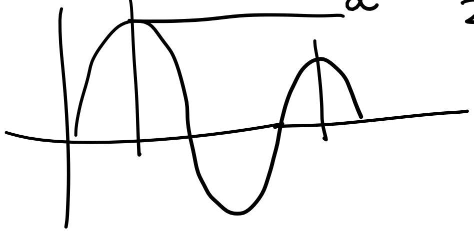
$\dot{x}(0) = v_0 = -\gamma A + B = 0$

$B = \gamma x_0$

$$= x_0 (1 + \gamma t) e^{-\gamma t}$$

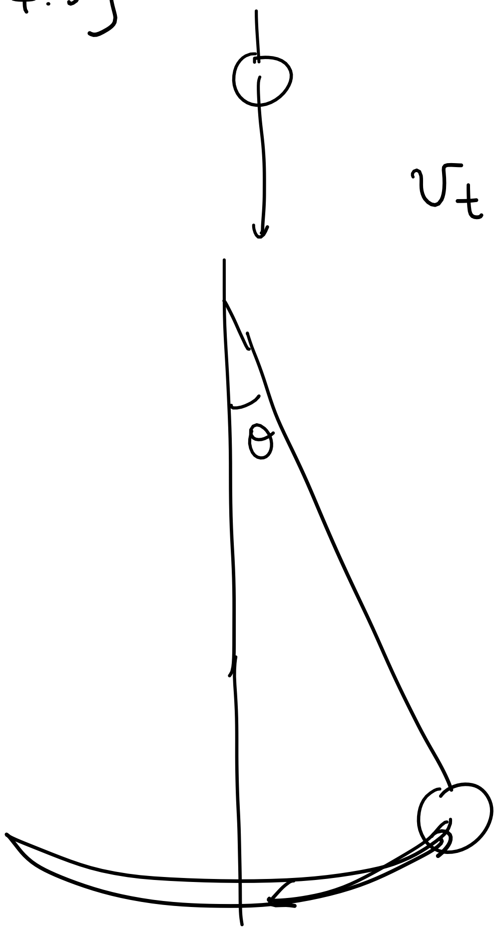
$$x(1) = ? = x_0 \underbrace{(1 + 2\pi) e^{-2\pi}} = x_0 \cdot (0.0136)$$

[Ex. 3.4.2] $\omega_d = \frac{1}{2}\omega_0 = \sqrt{\omega_0^2 - \gamma^2} \rightarrow \frac{\omega_0^2}{4} = \omega_0^2 - \gamma^2 \rightarrow \gamma = \omega_0 \frac{\sqrt{3}}{2}$



$$e^{-\gamma T_d} = e^{-\omega_0 \frac{\sqrt{3}}{2} \cdot \frac{2\pi}{\frac{\omega_0}{2}}} = e^{-2\sqrt{3}\pi} = 0.00002$$

[Ex 3.4.3]



$$v_t = \frac{mg}{c_1} = 30\,m/s \longrightarrow c_1 = \frac{mg}{30}$$

$$c_1 \dot{x} \longrightarrow \gamma = \frac{c_1}{2m} = \frac{\cancel{m}g}{2\cancel{m}\cdot 30} = \frac{g}{60} = 0.163$$

$$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{\frac{g}{l} - \gamma^2}} = \frac{2\pi}{\sqrt{\pi - (0.163)}} = 2.00268$$

$$(ex)\quad \frac{g}{l} = \pi^2 \qquad \sqrt{\frac{g}{l}} = \pi \rightarrow T_0 = 2\pi\sqrt{\frac{l}{g}} = 2$$

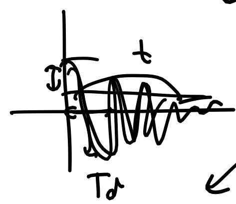
[Ex.3.4.4] 유체역학.

점성도 (viscosity) $\eta = 10^{-3}\,Ns/m^2$

$$C = 6\pi\eta r = 5\times10^{-5}\,Ns/m \qquad r = 0.00265\,m$$

$$\underline{\gamma = \frac{c}{2m}}, \quad \tau = \frac{1}{2\gamma} = \frac{m}{c} \qquad \omega_0^2 = \frac{k}{m} = 100, \quad k = 0.5N/m$$

$$Q = \frac{2\pi}{T_d/\tau} \simeq 100$$

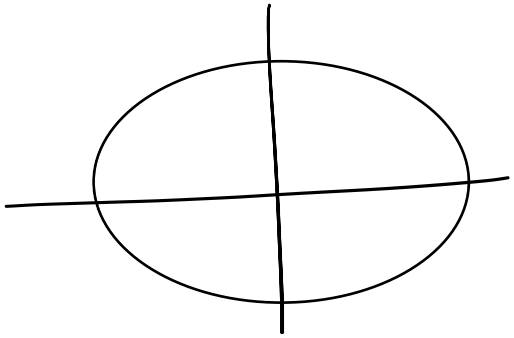


$$\frac{A}{A_0} = e^{-\frac{t}{2\tau}} = \frac{1}{2} \longrightarrow t = 2\tau \ln 2$$

$$n = \frac{t}{T_d} = \frac{2\ln 2}{(T_d/\tau) = \frac{2\pi}{Q}} = \frac{Q}{\pi}\ln 2 = 22.$$

Phase space

no damping

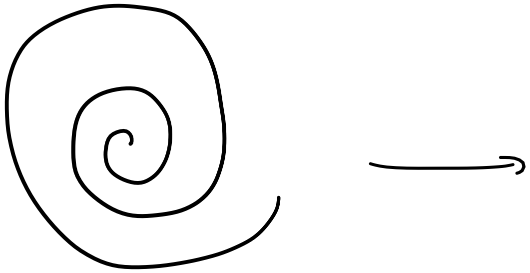


$$x = A\sin(\omega_0 t + \phi_0)$$

$$\dot{x} = v = \omega_0 A\cos(\omega_0 t + \phi_0)$$

$$\left(\frac{x}{A}\right)^2 + \left(\frac{v}{\omega_0 A}\right)^2 = 1$$

under damped.



[Ex. 3.5.2] nonlinear → chaos (혼돈이론)

$$F = \underline{\underline{x}} - x^3 \rightarrow \ddot{x} + \underbrace{2\gamma}_{1}\dot{x} = x - x^3$$

(자습)

$$\ddot{x} + \dot{x} - x + x^3 = 0$$

$x_0 = -1,\ v_0 = 1.4$	$x_0 = -1,\ v_0 = 1.45$	$x_0 = 0.01 \quad v_0 = 0$

→ 한계선 예.

3.6. Forced H.O.

$m\ddot{x} + kx + c\dot{x} = 0$ = "homogeneous"

$\neq 0$ = "in ″ ″"

↑

external force

$$m\ddot{x} + kx + c\dot{x} = F(t) = F_0 \cos\omega t$$

$c = 0$

$$m\ddot{x} + kx = F_0 \cos\omega t$$

양수

$x = A \cos\omega t$ 라고 가정

$\dot{x} = -A\omega \sin\omega t$

$\ddot{x} = -A\omega^2 \cos\omega t$

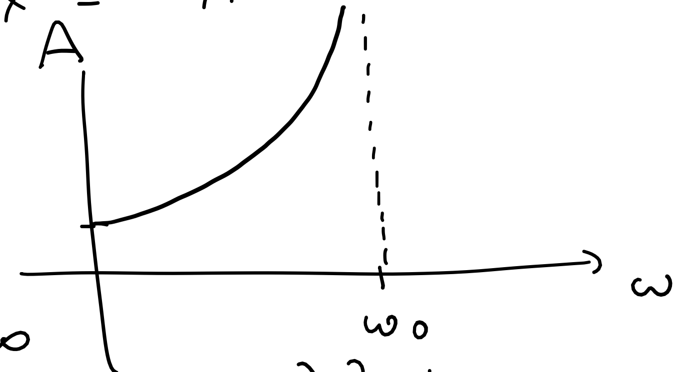
$\rightarrow (-mA\omega^2 + kA)\cancel{\cos\omega t} = F_0 \cancel{\cos\omega t}$

$$A = \frac{F_0/m}{\omega_0^2 - \omega^2}$$

$\omega < \omega_0$

$\omega > \omega_0$, $A < 0$

$\omega_0 > \omega$



$\omega \rightarrow \omega_0$ $A \rightarrow \infty$

(resonance) 공명.

더 일반적 해: $x = A\cos(\omega t - \phi)$

$$m\ddot{x} + kx = F(t) = \boxed{F_0 \cos\omega t}$$

$$= (-mA\omega^2 + kA)\cos(\omega t - \phi)$$

$$[\cos\omega t \cos\phi + \sin\omega t \sin\phi]$$

$$\sin\omega t\sin\phi = 0 \rightarrow \sin\phi = 0 \qquad \phi = 0, \pi$$

$$A(k - m\omega^2)\cos\phi = F_0$$

$$A = \frac{F_0/m}{\omega_0^2 - \omega^2} \cdot \frac{1}{\cos\phi}$$

if $\omega_0 > \omega \rightarrow \phi = 0 \longrightarrow x = \dfrac{F_0/m}{\omega_0^2 - \omega^2}\cos(\omega t)$

if $\omega > \omega_0 \rightarrow \phi = \pi \qquad A = \dfrac{F_0/m}{\omega^2 - \omega_0^2} \rightarrow x = \dfrac{F_0/m}{\omega^2 - \omega_0^2}\cos(\omega t - \pi)$

위상차

$c \neq 0$

$$m\ddot{x} + c\dot{x} + kx = F(t) = F_0 \cos\omega t = \underset{\text{실수부}}{\underline{\underline{Re}}}\,(F_0 \underbrace{e^{i\omega t}}_{\cos\omega t + i\sin\omega t})$$

$$Re[\,\underbrace{m\ddot{x} + c\dot{x} + kx = F_0 e^{i\omega t}}\,] \rightarrow Re(x)$$

$$= m\,\ddot{Re}(x) + c(\dot{Re}\,x) + k\,\underline{\underline{(Re\,x)}}$$

$$x(t) = A e^{i(\omega t - \phi)} \qquad (A > 0)$$

$$\dot{x} = i\omega A e^{i(\omega t - \phi)}$$

$$\ddot{x} = -\omega^2 A e^{i(\omega t - \phi)}$$

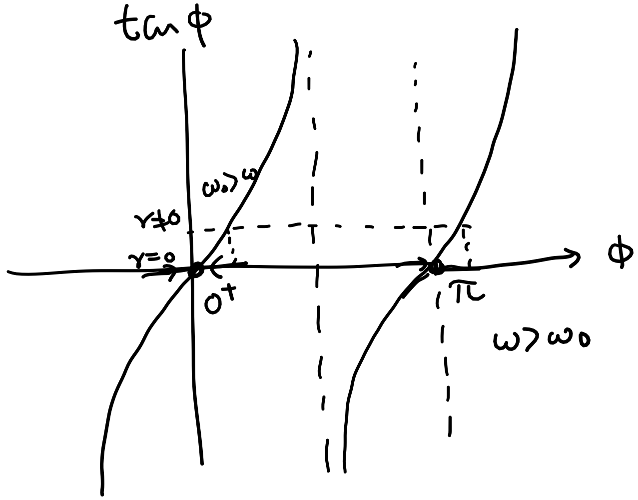
$$[-m\omega^2 + i c\omega + k]\, A e^{i(\cancel{\omega t} - \phi)} = F_0 \cancel{e^{i\omega t}}$$

$$e^{-i\phi} = \cos\phi - i\sin\phi$$

실수

$$A[(k - m\omega^2)\cos\phi + c\omega \sin\phi] = F_0$$

$$c\omega\cos\phi - (k - m\omega^2)\sin\phi = 0 \Rightarrow \tan\phi = \frac{c\omega}{k - m\omega^2} = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$



$$A\left[(k-m\omega^2)\cos\phi + c\omega\sin\phi\right] = F_0 \quad \rightarrow$$

$$\tan\phi = \frac{2\gamma\omega}{\omega_0^2-\omega^2} \qquad A = \frac{F_0/m}{\left[(\omega_0^2-\omega^2)\cos\phi + 2\gamma\omega\sin\phi\right]}$$

$$\left.\begin{aligned}\cos\phi &= B\times(\omega_0^2-\omega^2)\\ \sin\phi &= B\times 2\gamma\omega\end{aligned}\right\} \quad B^2 = \frac{1}{(\omega_0^2-\omega^2)^2+(2\gamma\omega)^2}$$

$$A = \frac{F_0/mB}{\left[(\omega_0^2-\omega^2)\underbrace{\cos\phi/B}_{\omega_0^2-\omega^2} + 2\gamma\omega\underbrace{\sin\phi/B}_{2\gamma\omega}\right]} = \frac{1}{B}\frac{F_0/m}{(\omega_0^2-\omega^2)^2+(2\gamma\omega)^2}$$

$$= \frac{F_0/m}{(\omega_0^2-\omega^2)^2+(2\gamma\omega)^2}\sqrt{(\omega_0^2-\omega^2)^2+(2\gamma\omega)^2}$$

$$= \frac{F_0/m}{\sqrt{(\omega_0^2-\omega^2)^2+(2\gamma\omega)^2}}$$

Resonance frequency

$(\omega_0^2-\omega^2)^2+(2\gamma\omega)^2$ 의 최소값 ; $\omega^2 \equiv y$; $(\omega_0^2-y)^2+4\gamma^2 y$

$\downarrow$

$-2(\omega_0^2-y)+4\gamma^2=0$

$y=\omega_0^2-2\gamma^2=\omega_r^2 \geq 0$

↑공명진동수

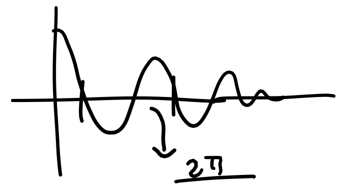
$\underline{\omega_0^2 \geq 2\gamma^2}$

$\downarrow$

$$(2\gamma^2)^2+4\gamma^2(\omega_0^2-2\gamma^2)$$

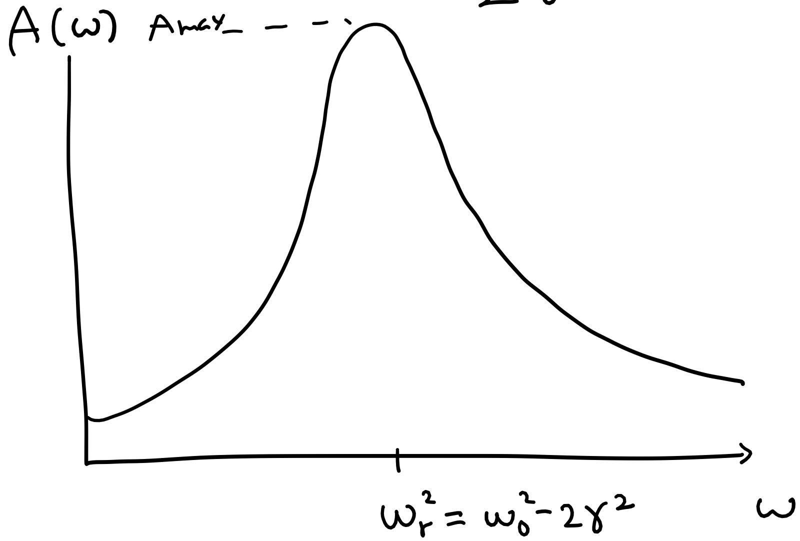
$$= 4\gamma^2(\omega_0^2-\gamma^2)$$

$$\therefore\ A_{max}=\frac{F_0/m}{2\gamma\sqrt{\omega_0^2-\gamma^2}}=\frac{F_0/m}{2\gamma\omega_d}$$

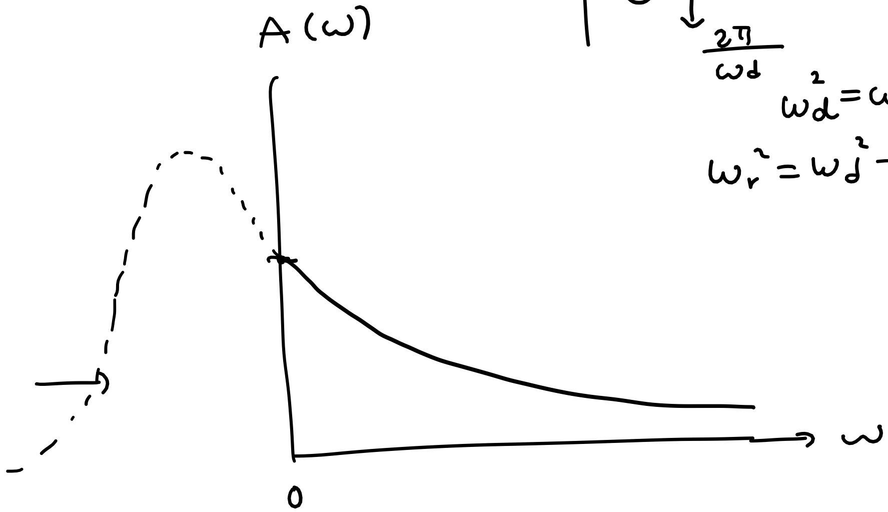


$\omega_d^2=\omega_0^2-\gamma^2$

$\omega_r^2=\omega_d^2-\gamma^2$



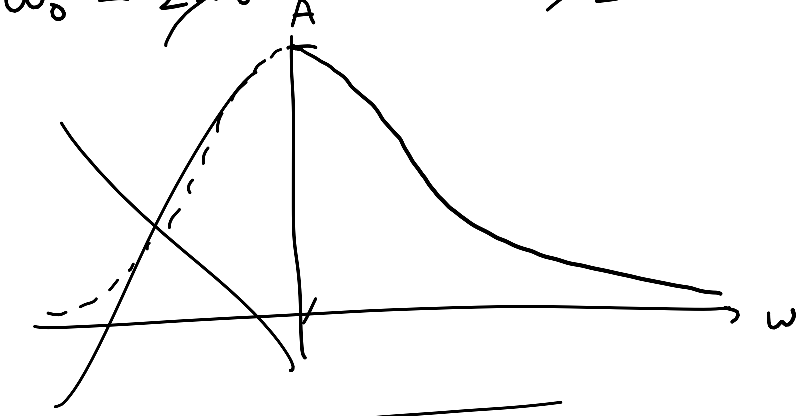
$\omega_0^2 \geq 2\gamma^2$



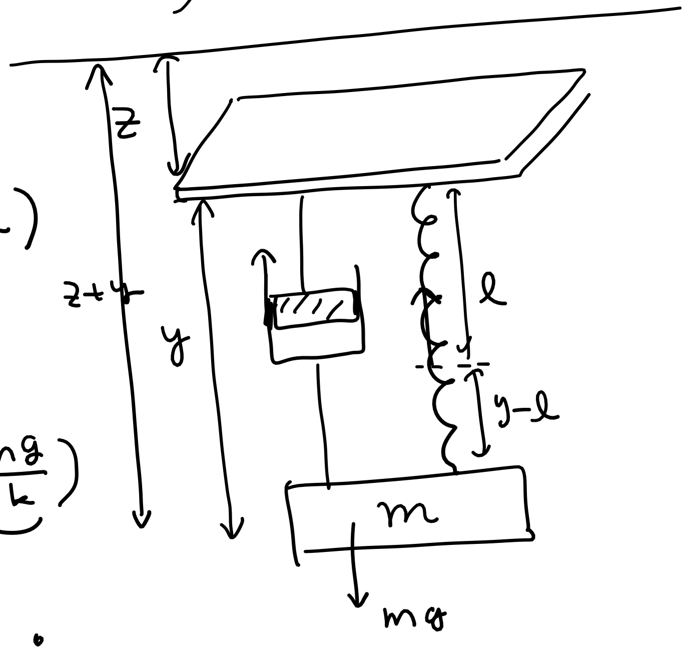
$$\gamma = \frac{\omega_0}{\sqrt{2}} \qquad \gamma^2 = \frac{\omega_0^2}{2} \longrightarrow \omega_r = 0$$

$$(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2 = \omega_0^4 - \cancel{2\omega_0^2\omega^2} + \omega^4 + 4\cancel{\frac{\omega_0^2}{2}}\omega^2 = \omega_0^4 + \omega^4$$

$$A = \frac{F_0/m}{\sqrt{\omega^4 + \omega_0^4}}$$



[Ex 3.6.1] 지진계



$$F = mg - c\dot{y} - k(y - l)$$

$$= m(\ddot{z} + \ddot{y})$$

$$= -c\dot{y} - k(\underbrace{y - l - \frac{mg}{k}}_{\equiv\, x})$$

$$= -c\dot{x} - kx \qquad \dot{x} = \dot{y}$$

$$= m(\ddot{z} + \ddot{x}) \qquad \ddot{x} = \ddot{y}$$

$$z = De^{i\omega t}$$

$$\longrightarrow m\ddot{x} + c\dot{x} + kx = -m\ddot{z} = \underbrace{m\omega^2 D}_{= F_0} e^{i\omega t}$$

$$\therefore\ A = \underbrace{\frac{\omega^2 D}{\sqrt{(\omega_0^2-\omega^2)^2+(2\gamma\omega)^2}}} = \frac{D}{\sqrt{\left(\frac{\omega_0^2}{\omega^2}-1\right)^2+\frac{4\gamma^2}{\omega^2}}}$$

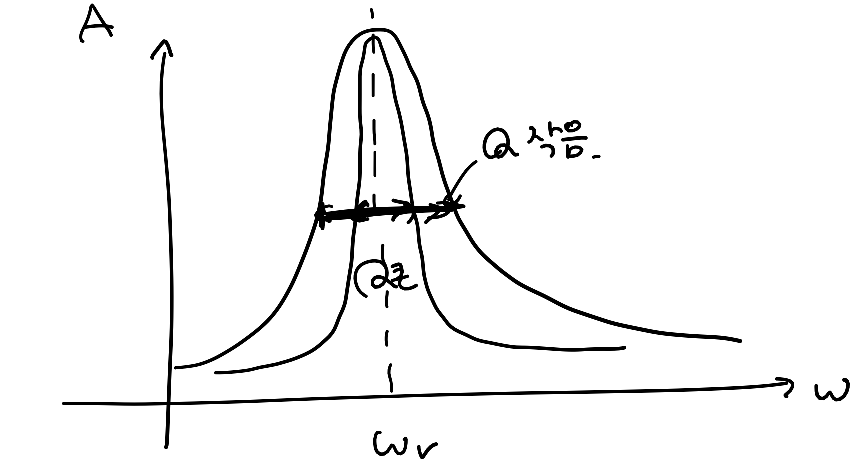
$$\underbrace{\frac{\omega_0^2}{\omega^2}<1}\ ,\ \frac{\gamma^2}{\omega^2}<1 \qquad \left(\ \underline{\gamma^2=\frac{\omega_0^2}{2}}\ \right)$$

$$A = \frac{D}{\sqrt{\underbrace{\left(\frac{\omega_0^2}{\omega^2}-1\right)^2+\frac{2\omega_0^2}{\omega^2}}_{1+\underbrace{\left(\frac{\omega_0^2}{\omega^2}\right)^2}_{\epsilon}}}} \cong D\left(1-\frac{1}{2}\left(\frac{\omega_0^2}{\omega^2}\right)^2\right) \quad \left\{\ \frac{1}{\sqrt{1+\epsilon}} \approx 1-\frac{\epsilon}{2}\right.$$

$\omega_0 = 2\pi\cdot 8\,Hz$

$$\frac{D-A}{D} = \frac{D-D\left(1-\frac{1}{2}(\ )^2\right)}{D} = \underline{\frac{1}{2}\frac{\omega_0^4}{\omega^4}} \cong \frac{1}{10} \longrightarrow \omega_0 = 0.84\,\omega$$

$\gamma = \frac{\omega_0}{\sqrt{2}} = 36$



Quality factor

$$\underbrace{\gamma \ll \omega_0}_{} \rightarrow \omega_0 \approx \omega_d \approx \omega_r \qquad \underbrace{\begin{aligned}\omega_d^2 &= \omega_0^2 - \gamma^2 \\ \omega_r^2 &= \omega_0^2 - 2\gamma^2\end{aligned}}_{}$$

$$\omega_0^2 - \omega^2 = (\omega_0 + \omega)(\omega_0 - \omega)$$

$$\underset{\omega \approx \omega_0}{\approx} 2\omega_0(\omega_0 - \omega)$$

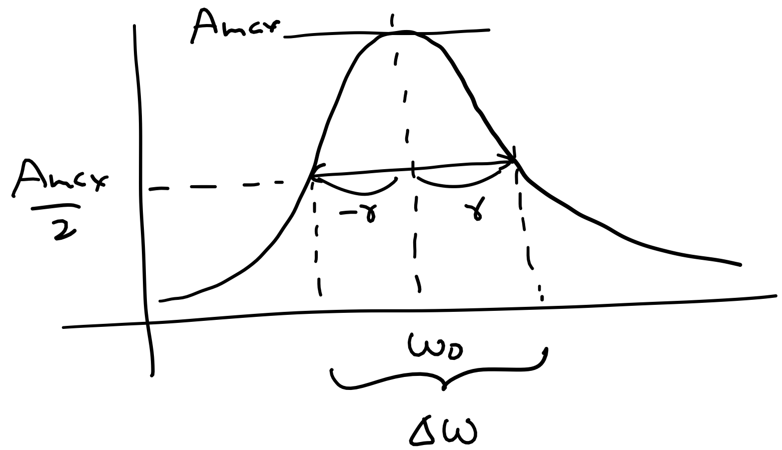
$$\omega = \omega_0 + \epsilon$$

$$-(2\omega_0 + \epsilon)\epsilon \simeq -2\omega_0\epsilon - \cancel{\epsilon^2} = 2\omega_0(\omega_0 - \omega)$$

$$4\gamma^2\omega^2 \approx 4\gamma^2\omega_0^2$$

$$A \cong \frac{A_{max} \cdot \gamma}{\sqrt{4\omega_0^2(\omega_0 - \omega)^2 + 4\omega_0^2\gamma^2}} = \frac{A_{max}\,\gamma / 2\omega_0}{\sqrt{(\omega - \omega_0)^2 + \gamma^2}}$$

$$A = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}}$$



$$Q = \frac{\omega_0}{2\gamma}$$

$$2\gamma = \frac{\omega_0}{Q} = \Delta\omega \rightarrow \frac{\Delta\omega}{\omega_0} = \frac{1}{Q}$$

Phase

$$\phi = \tan^{-1}\left(\frac{2\gamma\omega}{\omega_0^2-\omega^2}\right)$$