1학기 역학 내용

1. vector
2. 직선운동
3. sine, cosine (조화운동)
4. 2, 3 차원 운동 (→ 일차원)
5. 비관성 좌표계
6. 중심력 운동 (일차원)

좌표가 1개. (일차원)

1개의 자유도.

2학기: 2개.

1 장.

역학 : $g(t)$ ← 물리량 : (ex) 속도, 위치, 에너지, ... } ~~전하~~

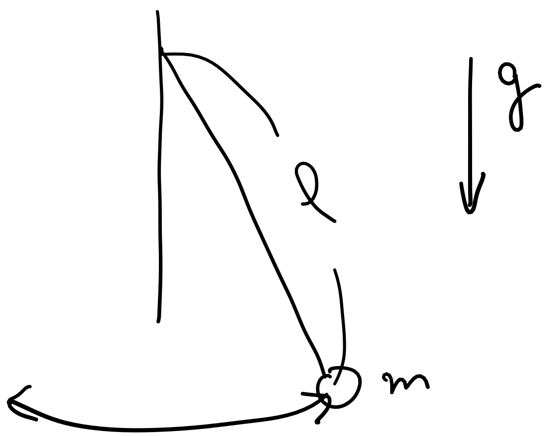
	시간 (T)	길이 (L)	질량 (M)	
단위	초 (s)	m	kg	mks

단위가 있는 양 : (ex) 속도 : m/초 $\qquad [\text{속도}] = \dfrac{L}{T}$

〃 없는 양 : $\left(\text{비} : \dfrac{M}{M_{\odot}} \right)$

$\odot$ ← 태양

차원 해석 (Dimensional analysis)



$$[\text{중력가속도}] = \frac{\frac{L}{T}}{T} = LT^{-2}$$

$\alpha = 0, \ \gamma = -\frac{1}{2}, \ \beta = \frac{1}{2}$

$\Uparrow$

$\alpha = 0 \qquad \beta + \gamma = 0$

$-2\gamma = 1$

$$\tau = m^{\alpha} l^{\beta} g^{\gamma} = l^{\frac{1}{2}} g^{-\frac{1}{2}} = \sqrt{\frac{l}{g}}$$

$$T = M^{\alpha} L^{\beta} (LT^{-2})^{\gamma} = M^{\alpha} L^{\beta+\gamma} T^{-2\gamma}$$

Vector

(a, b, c) $\quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \rightarrow 3$ 차원

$\downarrow$

n 〃 : $(a_1, a_2, \cdots, a_n)$

$+ \quad a_1, a_2, \cdots a_n$; 동일한 차원.

(ex) $(50kg, 160cm, 21yr)$; X

} 함수 : $e^{\cancel{(10kg)}}$, $\log(\cancel{1\,sec})$ ...
X

$$e^{x} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

차원X

$\uparrow$ 차원x, $\uparrow$ M, $\uparrow$ M^2, $\uparrow$ M^3 ...

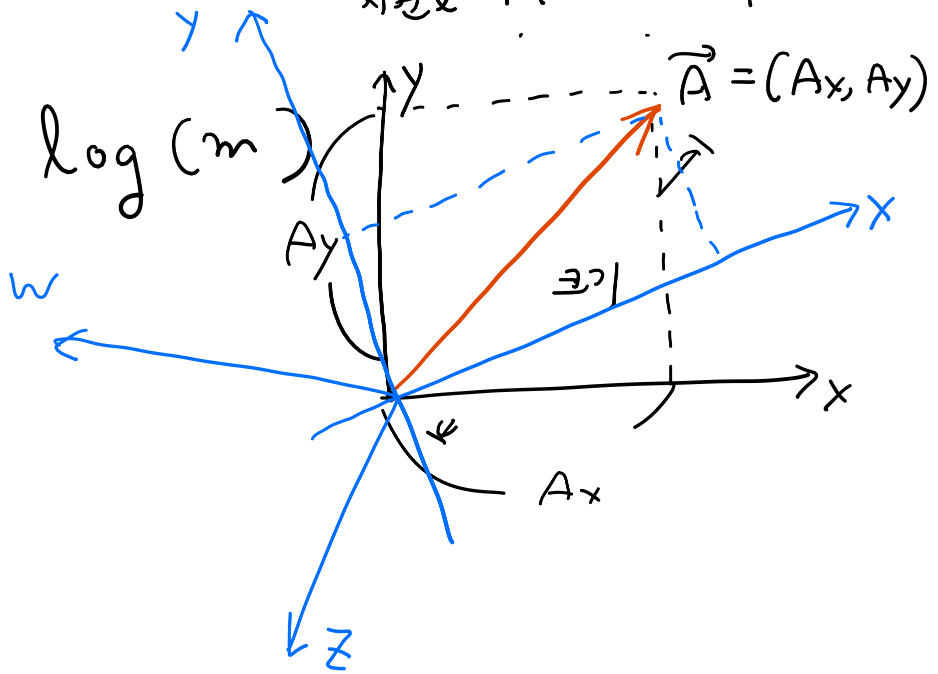
(ex) 틀린 예

$$\tau = \log(m)$$

$\vec{A}$: 크기와 방향.

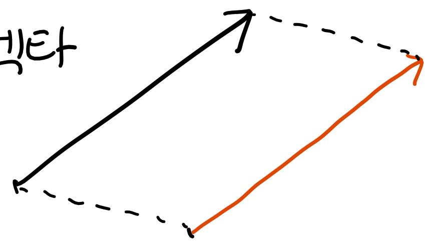
$$\underbrace{\vec{A}}_{\text{축 이전에 정의됨}} \text{"="} \underbrace{(A_x, A_y, A_z)}_{\text{축(axis)에 대한 성분}}$$

성분



동등성 : 크기와 방향이 같으면 동일

1차원 벡타

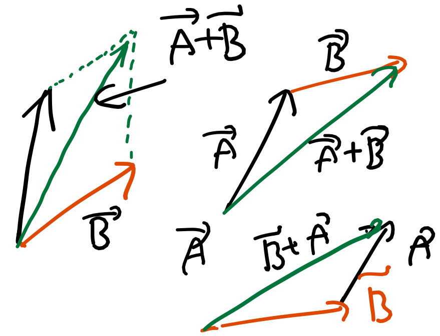


스칼라 : 크기만. (숫자)

↕

벡타의 연산 :

① 덧셈 (n)차원이 같은 벡타끼리.



성분(축) $\vec{A} = (A_x, A_y, A_z)$

$\vec{B} = (B_x, B_y, B_z)$

$\vec{B}+\vec{A} = \vec{A}+\vec{B} = (A_x+B_x, A_y+B_y, A_z+B_z)$

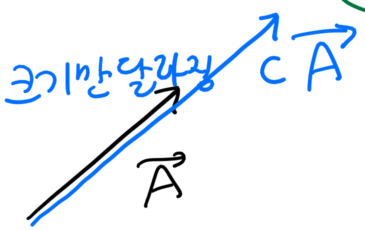
$A_x+B_x = B_x+A_x$

교환법칙

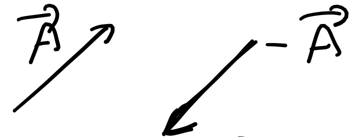
결합 " $(\vec{A}+\vec{B})+\vec{C} = \vec{A}+(\vec{B}+\vec{C})$

② 곱셈 $\begin{cases} 스 \times 벡 \\ 벡 \times 벡 = \begin{cases} 스 \\ 벡 \end{cases} \end{cases}$ (나중)

$\vec{A}$: $c\vec{A}$: 크기만 달라짐

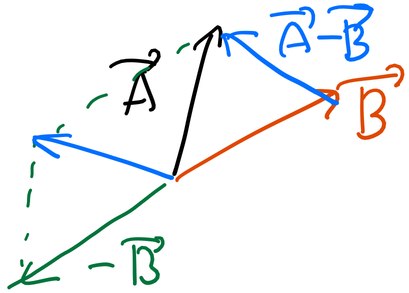


특히 $c = -1$; $-\vec{A}$



③ 뺄셈 : $\vec{A}-\vec{B} = \vec{A} + (-1)\cdot\vec{B}$

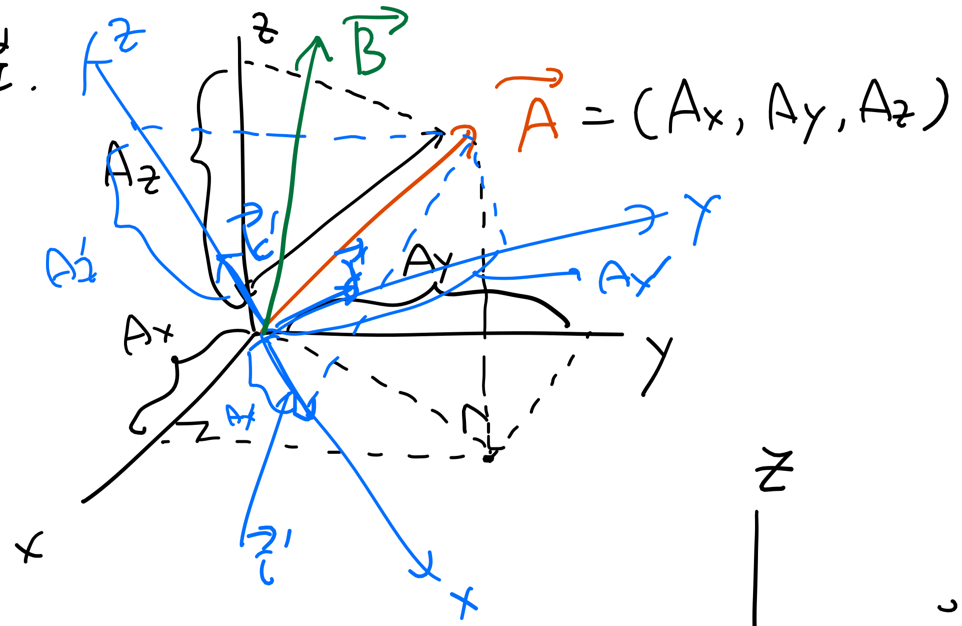
④ 나눗셈 : $\dfrac{벡}{스}$

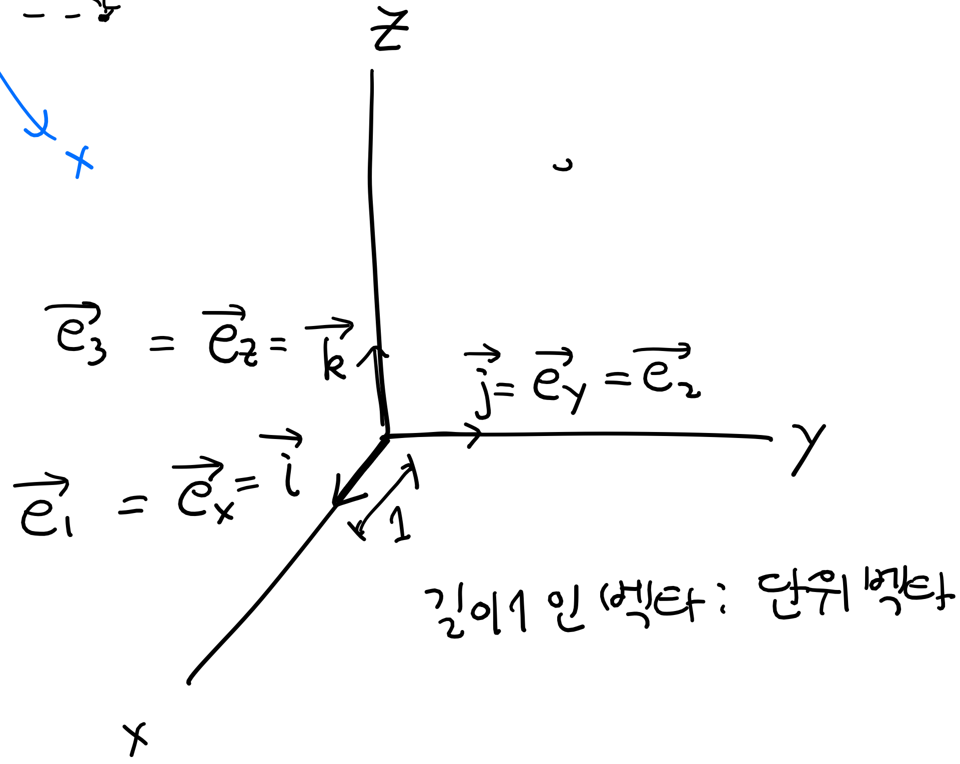


$\dfrac{1}{스} = 스$

$\vec{A}$ vector 크기를 성분.

$$|\vec{A}| = \sqrt{A_x^2+A_y^2+A_z^2} = \sqrt{A_x'^2+A_y'^2+A_z'^2}$$





길이1 인 벡터 : 단위 벡터

✓

$$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$

간단히 표현

$$= (A_x, A_y, A_z)$$

$$= A_x'\vec{i}' + A_y'\vec{j}' + A_z'\vec{k}'$$

$$\vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k}$$

$$= B_x'\vec{i}' + B_y'\vec{j}' + B_z'\vec{k}'$$

$$\vec{A}+\vec{B} = (A_x+B_x)\vec{i} + \cdots$$

$$= (A_x'+B_x')\vec{i}' + (A_y'+B_y')\vec{j}' + (A_z'+B_z')\vec{k}'$$

곱 : 벡 × 벡 = $\begin{cases} \text{스} \\ \text{벡} \end{cases}$ ○

Scalar 곱 "·"

성분

$$\vec{A} \cdot \vec{B} = \underline{\text{스}} = A_x B_x + A_y B_y + A_z B_z$$

"내적 : inner product"

$$= A'_x B'_x + A'_y B'_y + A'_z B'_z$$

축과 무관하게 똑 같다.

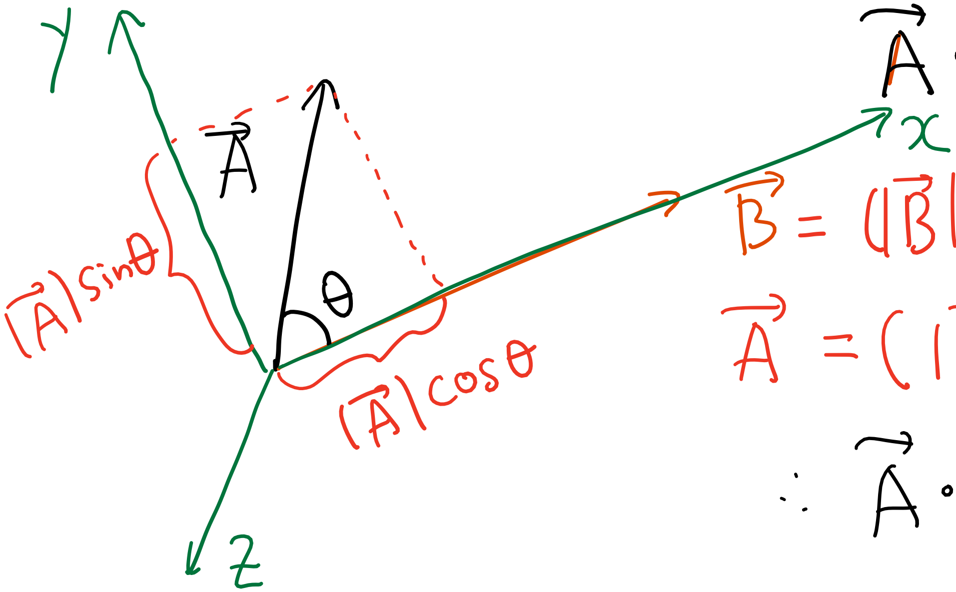
3차원

n차원 $\vec{A} = (A_1, \cdots, A_n)$

$\vec{B} = (B_1, \cdots, B_n)$

$$\vec{A} \cdot \vec{B} = A_1 B_1 + \cdots + A_n B_n = \sum_{i=1}^{n} A_i B_i$$

$$\underline{\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta}$$



$\vec{B} = (|\vec{B}|, 0, 0)$

$\vec{A} = (|\vec{A}|\cos\theta, |\vec{A}|\sin\theta, 0)$

$\therefore \vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$ ✓

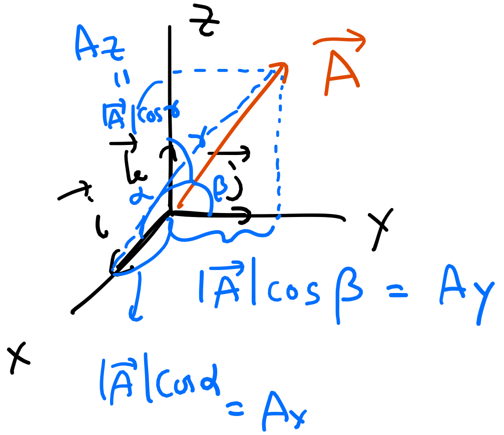
$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|}$$

$$\vec{A}\cdot\vec{A} = |\vec{A}|\,|\vec{A}| \Rightarrow |\vec{A}| = \sqrt{\vec{A}\cdot\vec{A}}$$

$\vec{A} = \vec{B}$

$\vec{A} \perp \vec{B}$

$\theta = \frac{\pi}{2}$ $\quad \cos\frac{\pi}{2} = 0 \quad \rightleftarrows \quad \vec{A}\cdot\vec{B} = 0$



$$\vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{k} = \vec{i}\cdot\vec{k} = 0, \quad \vec{i}\cdot\vec{i} = 1$$

$$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$

$$= |\vec{A}|\underbrace{\left(\cos\alpha\,\vec{i} + \cos\beta\,\vec{j} + \cos\gamma\,\vec{k}\right)}_{=\vec{n}}$$

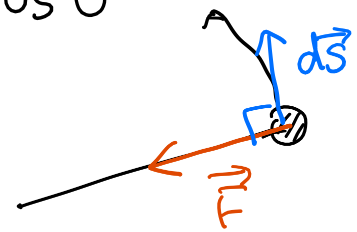
단위벡터

$$|\vec{A}| = |\vec{A}|\,\underbrace{|\vec{n}|}_{1}$$

$$|\vec{n}| = 1 = \sqrt{\underbrace{\cos^2\alpha + \cos^2\beta + \cos^2\gamma}_{1}}$$

Ex. 1.4.1. 스칼라 곱 : Work (일)

$$W = \underset{\substack{\uparrow \\ \text{힘}}}{\vec{F}} \cdot \underset{\substack{\uparrow \\ \text{움직인 거리}}}{\Delta\vec{s}} = |\vec{F}||\Delta\vec{s}|\cos\theta$$



$\rightarrow W = 0$

1.5. Vector 곱 : 벡 x 벡 = $\underbrace{\text{벡}}$: 3차원 벡터 only!

$$\vec{A} \times \vec{B} \overset{\text{성분}}{=} (\underbrace{A_y B_z - A_z B_y}_{x\text{-성분}}, \underbrace{A_z B_x - A_x B_z}_{y\text{-성분}}, \underbrace{A_x B_y - A_y B_x}_{z\text{-성분}})$$

M : 3x3 행렬 $\rightarrow$ determinant (스칼라)

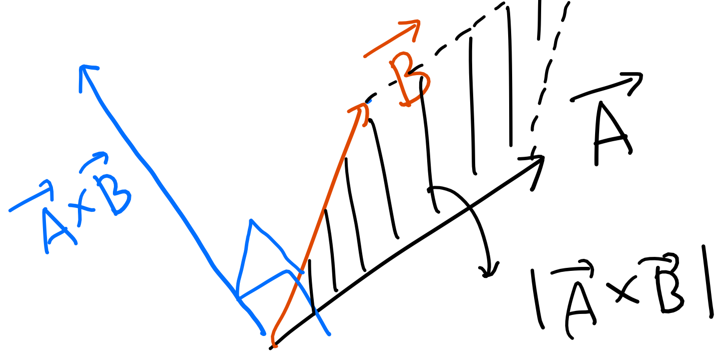
$|M|$

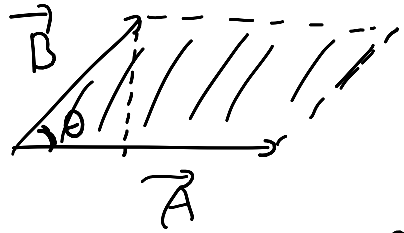
$$\begin{vmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{vmatrix}$$

$2\times 2 \begin{pmatrix} a & b \\ c & d \end{pmatrix} \rightarrow ad - bc$

$$= M_{11}(M_{22}M_{33} - M_{23}M_{32})$$
$$- M_{12}(M_{21}M_{33} - M_{23}M_{31})$$
$$+ M_{13}(M_{21}M_{32} - M_{22}M_{31})$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \vec{i}\,(A_yB_z - A_zB_y) - \vec{j}\,(A_xB_z - A_zB_x) + \vec{k}\,(A_xB_y - A_yB_x)$$

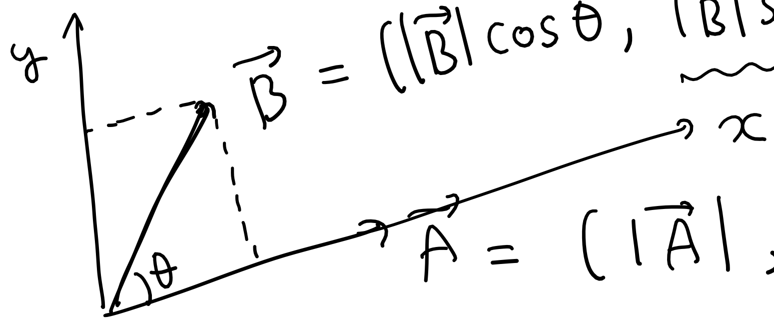




$$|\vec{A}\times\vec{B}| = |\vec{A}|\,|\vec{B}|\sin\theta$$

$$\vec{A}\times\vec{B} = -\,\vec{B}\times\vec{A}$$

(pf)



$\vec{B} = (|\vec{B}|\cos\theta,\ |\vec{B}|\sin\theta,\ 0)$

$\vec{A} = (|\vec{A}|,\ 0,\ 0)$

$\vec{A}\times\vec{B} = (0,\ 0,\ |\vec{A}||\vec{B}|\sin\theta)$

$= \vec{k}\,(\quad)$

$$|\vec{A}\times\vec{B}|^2 = (\vec{A}\times\vec{B})\cdot(\vec{A}\times\vec{B})$$

$$(|\vec{C}|^2 = \vec{C}\cdot\vec{C})$$

$$(a+b)^2 = a^2+b^2+2ab$$

$$= (A_yB_z - A_zB_y)^2 + (A_zB_x - A_xB_z)^2 + (A_xB_y - A_yB_x)^2$$

$$= A_y^2B_z^2 + A_z^2B_y^2 + A_z^2B_x^2 + A_x^2B_z^2 + A_x^2B_y^2 + A_y^2B_x^2 + A_x^2B_x^2 + A_y^2B_y^2 + A_z^2B_z^2$$

$$\underbrace{(A_x^2+A_y^2+A_z^2)}_{|\vec{A}|^2}\underbrace{(B_x^2+B_y^2+B_z^2)}_{|\vec{B}|^2}$$

$$-(A_x^2B_x^2 + A_y^2B_y^2 + A_z^2B_z^2)$$

$$-2A_yA_zB_yB_z - 2A_xA_zB_xB_z - 2A_xA_yB_xB_y$$

$$-(\underbrace{A_xB_x + A_yB_y + A_zB_z}_{\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta})^2$$

$$= |\vec{A}|^2|\vec{B}|^2 - (\vec{A}\cdot\vec{B})^2$$

$$= |\vec{A}|^2|\vec{B}|^2(1-\cos^2\theta) = |\vec{A}|^2|\vec{B}|^2\sin^2\theta$$

$$|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$$

✓

$$(\vec{A}\times\vec{B})\cdot\vec{A} = 0 = (\vec{A}\times\vec{B})\cdot\vec{B}$$

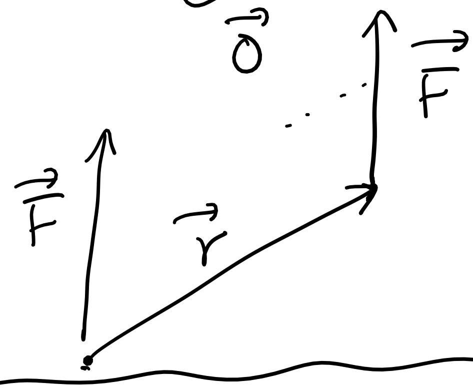
$\theta = 0 \rightarrow \sin\theta = 0$ $\qquad \vec{A}\times\vec{A} = 0$ $\qquad (\vec{A}\times\vec{B} = -\vec{B}\times\vec{A})$

$$\vec{A}\times\vec{A} = -\vec{A}\times\vec{A}$$

$$2\underbrace{(\vec{A}\times\vec{A})}_{\vec{0}} = \vec{0}$$

1.6. Vector (Cross) product

돌림힘 (torque) $\underline{\vec{N} = \vec{r}\times\vec{F}}$



1.7. 3개의 벡터 곱

· $(\vec{A}\times\vec{B})\times\vec{C}$ vs. $\vec{A}\times(\vec{B}\times\vec{C})$

$(\vec{A}\times\vec{B})\times\vec{C} = -\vec{C}\times(\vec{A}\times\vec{B})$

· $\vec{A}\cdot(\vec{B}\times\vec{C})$

$(\vec{A}\times\vec{B})\cdot\vec{C} = \vec{C}\cdot(\vec{A}\times\vec{B})$

$\vec{A}\cdot\vec{B}\cdot\vec{C}$ ×

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = A_x(B_yC_z - B_zC_y) - A_y(B_xC_z - B_zC_x) + A_z(B_xC_z - B_zC_x)$$

$$= \begin{vmatrix} C_x & C_y & C_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \qquad - \begin{vmatrix} B_x & B_y & B_z \\ A_x & A_y & A_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$= \begin{vmatrix} A_y & A_z & A_x \\ B_y & B_z & B_x \\ C_y & C_z & C_x \end{vmatrix}$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{C}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{C}\times\vec{A})$$

$$= -\vec{B}\cdot(\vec{A}\times\vec{C})$$

$$\boxed{\vec{A}\times(\vec{B}\times\vec{C}) = \text{벡터} = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})}$$

$$= \big(A_y(B_xC_y - B_yC_x) - A_z(B_zC_x - B_xC_z),\ \cdots,\ \cdots\big)$$

$$(2\vec{A} = \vec{A}2)$$

1.8. 좌표계의 변환

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

$$= A_x' \vec{i}' + A_y' \vec{j}' + A_z' \vec{k}'$$

$$\vec{A} \cdot \vec{i}' = A_x' = A_x \vec{i} \cdot \vec{i}' + A_y \vec{j} \cdot \vec{i}' + A_z \vec{k} \cdot \vec{i}'$$

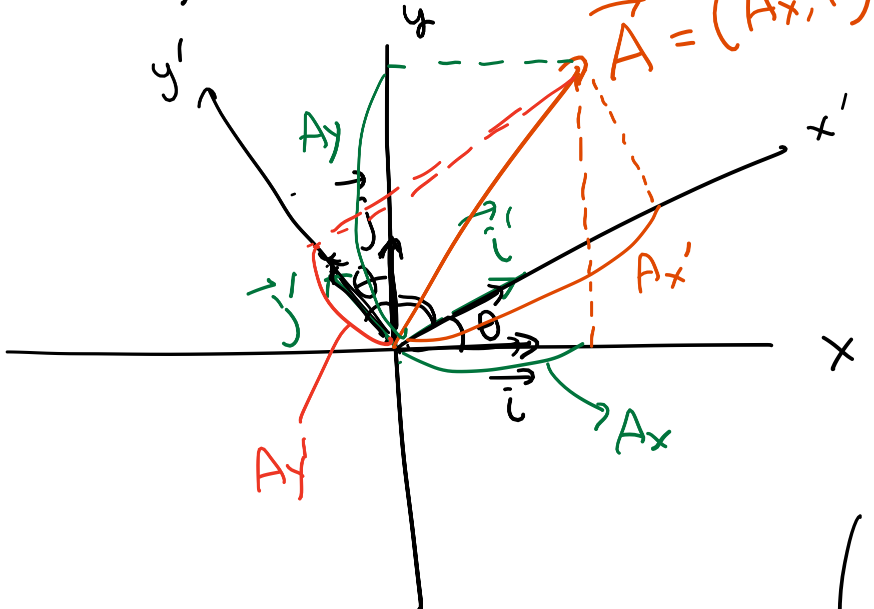
$$\vec{A} \cdot \vec{j}' = A_y' = A_x \vec{i} \cdot \vec{j}' + A_y \vec{j} \cdot \vec{j}' + A_z \vec{k} \cdot \vec{j}'$$

$$\vec{A} \cdot \vec{k}' = A_z' = A_x \vec{i} \cdot \vec{k}' + A_y \vec{j} \cdot \vec{k}' + A_z \vec{k} \cdot \vec{k}'$$

$$\begin{pmatrix} A_x' \\ A_y' \\ A_z' \end{pmatrix} = \underbrace{\begin{pmatrix} \vec{i} \cdot \vec{i}' & \vec{j} \cdot \vec{i}' & \vec{k} \cdot \vec{i}' \\ \vec{i} \cdot \vec{j}' & \vec{j} \cdot \vec{j}' & \vec{k} \cdot \vec{j}' \\ \vec{i} \cdot \vec{k}' & \vec{j} \cdot \vec{k}' & \vec{k} \cdot \vec{k}' \end{pmatrix}}_{\text{회전행렬} = R} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

(ex) z축에 대한 회전

$\vec{A} = (A_x, A_y, A_z)$



$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} A_x' \\ A_y' \\ A_z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$$\therefore A_x' = \cos\theta\, A_x + \sin\theta\, A_y$$

$$A_z' = A_z$$

$$AB \neq BA$$

$\theta = \frac{\pi}{2}$

$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \neq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$|\vec{A}|$ ← 좌표축과 무관.

$|\vec{A}| = |\vec{A}'|$

$\vec{A}' = R \cdot \vec{A}$

$|\vec{A}| = |R\vec{A}|$

↓

$\vec{A} \cdot \vec{A} = \vec{A}' \cdot \vec{A}' = \underline{A^T A = A'^T A'}$

A^T

column

(cf)

Transpose

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad M^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

$$N = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \qquad N^T = \begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \xrightarrow{T} \begin{pmatrix} a & b & c \end{pmatrix}$$

$$\underbrace{A^T A} = A'^T A' = (RA)^T (RA) = A^T \boxed{R^T R} A$$

$$R^T R = \mathbb{1}$$

$$A' = RA$$

$$\begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

$$(AB)^T = B^T A^T$$

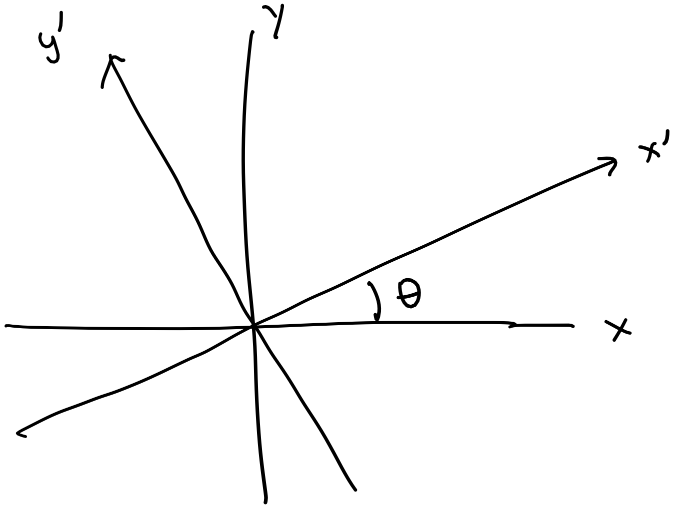
$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \qquad (\cdot \ \cdot \ \cdot) \qquad \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

(tensor → 3, 4, ⋯)

직교행렬 (orthogonal matrix)

$$R^T R = \mathbb{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

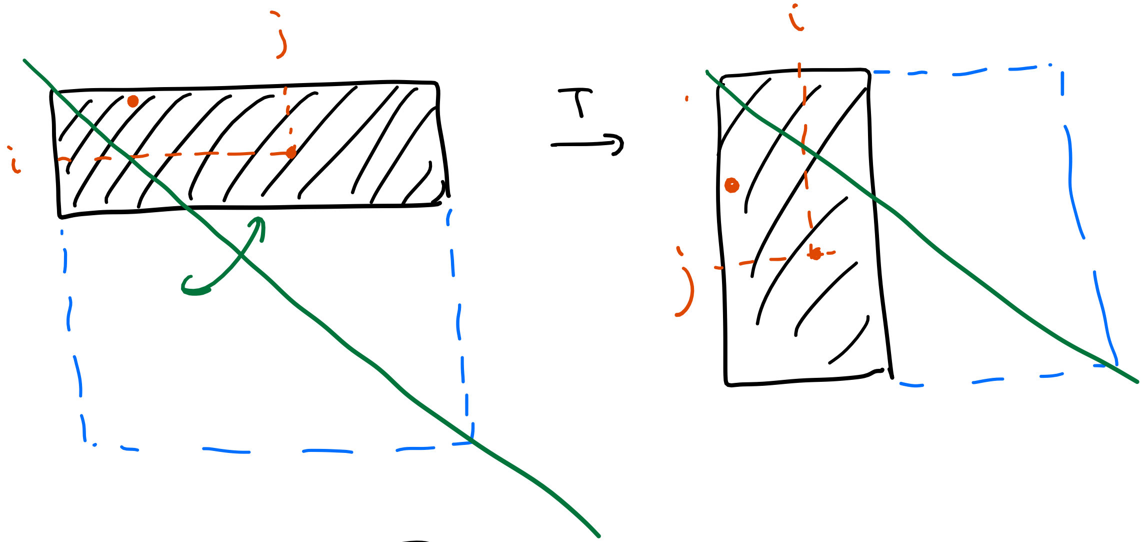
(ex)



$$R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$R^T = \begin{pmatrix} \cancel{\cos\theta} & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$R^T R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



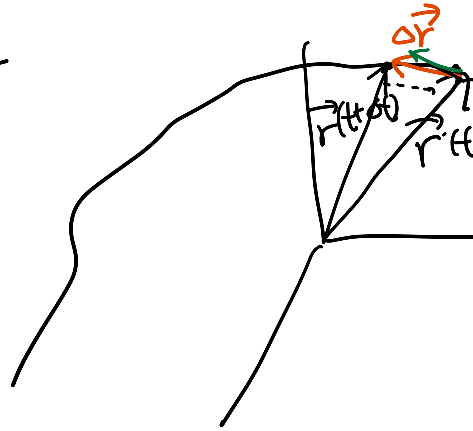
$$\begin{pmatrix} a & b & c \end{pmatrix} \xrightarrow{T} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & & \\ a_{mn} & \cdots & a_{mn} \end{pmatrix} \quad m \times n$$

$\downarrow$ Traspose

$$\begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & & a_{mn} \end{pmatrix} \quad n \times m$$

Vector 의 미분



$\vec r(t) = (\underbrace{x(t), y(t), z(t)})$

$$\vec V = \frac{d\vec r}{dt}$$

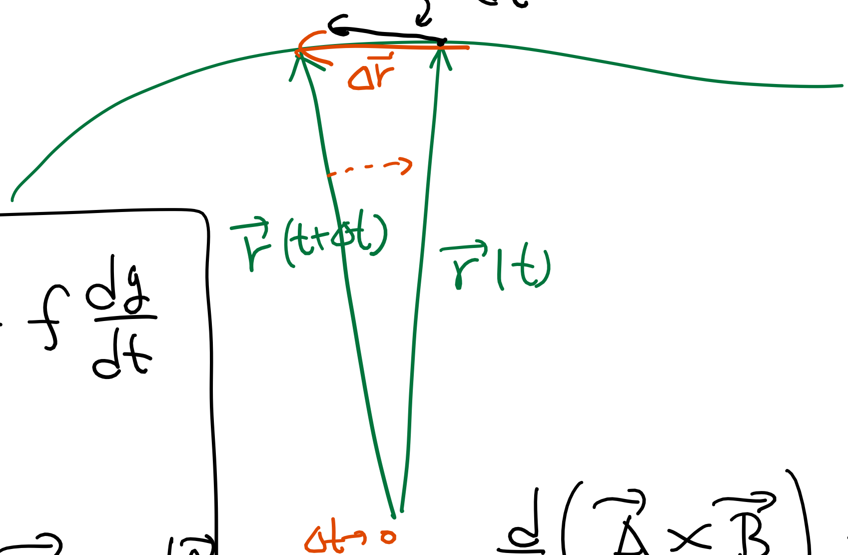
velocity (vector) 속도

speed (scalar) 속력

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$\lim_{\Delta t \to 0} \frac{\overbrace{\vec r(t+\Delta t) - \vec r(t)}^{=\Delta \vec r}}{\Delta t}$$

$$\frac{d\vec r}{dt} = \vec v = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$$



$$\frac{d}{dt}\left(f(t)\, g(t)\right) = \frac{df}{dt} g + f \frac{dg}{dt}$$

$$\frac{d}{dt}\left(f(t)\, \vec A(t)\right) = \frac{df}{dt}\vec A + f\frac{d\vec A}{dt}$$

$$\frac{d}{dt}\left(\vec A \times \vec B\right) = \frac{d\vec A}{dt} \times \vec B + \vec A \times \frac{d\vec B}{dt}$$

$$\frac{d}{dt}\left(\vec A \cdot \vec B\right) = \frac{d\vec A}{dt} \cdot \vec B + \vec A \cdot \frac{d\vec B}{dt}$$

	vec	scal
상수	①	②
함수	③.	④

(ex) $\underbrace{\vec{r} = (2t,\ t^2)}$ ← vector

$$\dot{\vec{r}} = (2, 2t)$$

$$\ddot{\vec{r}} = (0, 2)$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}}$$

$$\vec{a} = \frac{d\vec{v}}{dt} \leftarrow \text{acceleration}$$

$$= \frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}} = (\ddot{x}, \ddot{y}, \ddot{z})$$

$$= \vec{i}\,\ddot{x} + \vec{j}\,\ddot{y} + \vec{k}\,\ddot{z}$$

(Ex) $\vec{r}(t) = \underline{\underline{\vec{i}}}\, bt + \underline{\underline{\vec{j}}}\left(ct - \frac{gt^2}{2}\right) + \underline{\underline{\vec{k}}}\, 0$

$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{i}\, b + \vec{j}(c - gt) \longrightarrow \vec{a} = \frac{d^2\vec{r}}{dt^2} = -g\vec{j}$$

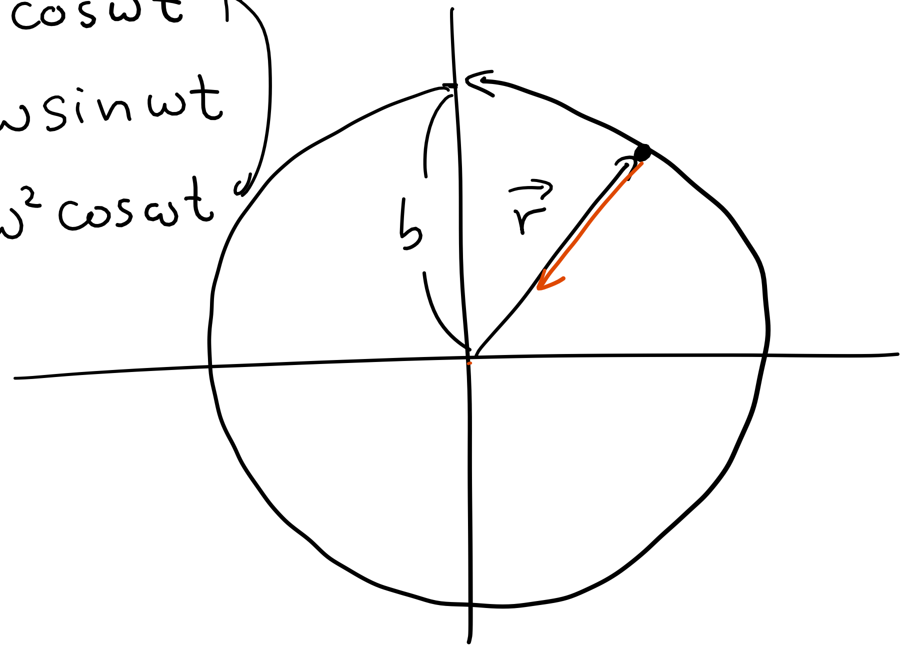
$$v = |\vec{v}| = \sqrt{b^2 + (c-gt)^2}$$

(ex) $\vec{r} = \vec{i}\, b\sin\omega t + \vec{j}\, b\cos\omega t$

$\vec{v} = \vec{i}\, b\omega\cos\omega t - \vec{j}\, b\omega\sin\omega t$

$\vec{a} = -\vec{i}\, b\omega^2\sin\omega t - \vec{j}\, b\omega^2\cos\omega t$

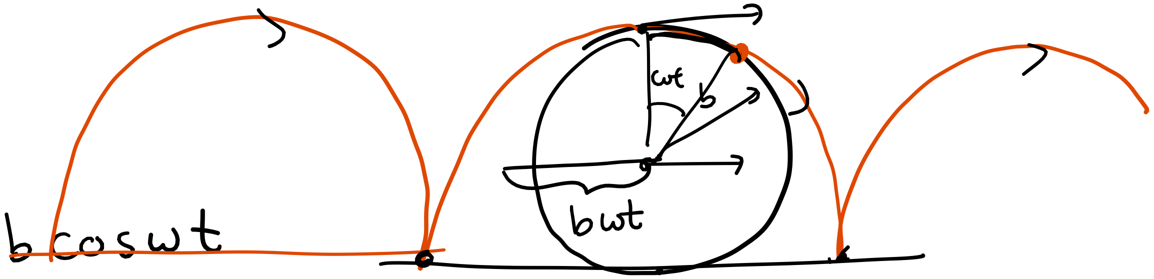
$= -\omega^2\vec{r}$



(ex) $\vec{r} = \vec{r}_1 + \vec{r}_2$

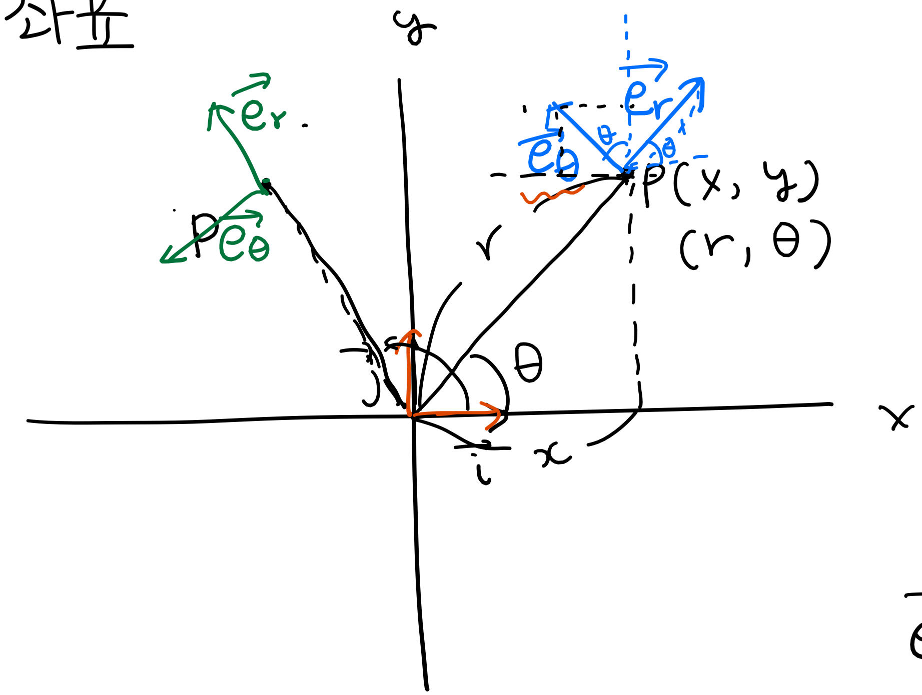
$\vec{r}_1 = \vec{i}\, b\omega t + \vec{j}\, b$

$\vec{r}_2 = \vec{i}\, b\sin\omega t + \vec{j}\, b\cos\omega t$



$\vec{v} = \vec{i}\,(b\omega + b\omega\cos\omega t) + \vec{j}\,(-b\omega\sin\omega t)$

11. 극좌표

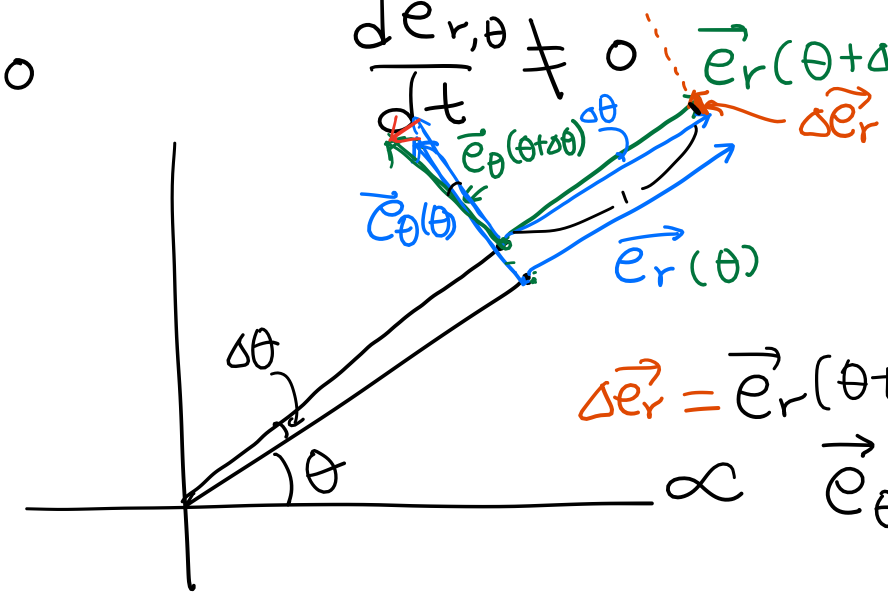


$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\vec{e_r} = \vec{i}\cos\theta + \vec{j}\sin\theta$$
$$\vec{e_\theta} = -\vec{i}\sin\theta + \vec{j}\cos\theta$$

$$\frac{d\vec{i}}{dt} = 0 \qquad \frac{d\vec{j}}{dt} = 0 \qquad \frac{d\vec{e}_{r,\theta}}{dt} \neq 0$$

$$\frac{d\vec{e_r}}{d\theta} = \lim_{\Delta\theta\to 0}\frac{\Delta\vec{e_r}}{\Delta\theta} = \lim_{\Delta\theta\to 0}\frac{1\cdot\cancel{\Delta\theta}\cdot\vec{e_\theta}}{\cancel{\Delta\theta}} = \vec{e_\theta}$$



$$|\Delta\vec{e_r}| = 1\cdot\Delta\theta$$

$$\Delta\vec{e_r} = \vec{e_r}(\theta+\Delta\theta) - \vec{e_r}(\theta) \propto \vec{e_\theta}$$

$$\Delta \vec{e_\theta} = \vec{e_\theta}(\theta+\Delta\theta) - \vec{e_\theta}(\theta) \propto -\vec{e_r}$$

$$|\Delta\vec{e_\theta}| = \Delta\theta$$

$$\Delta\vec{e_\theta} = -(\Delta\theta)\,\vec{e_r} \;\to\; \lim_{\Delta\theta\to 0}\frac{\Delta\vec{e_\theta}}{\Delta\theta} = \frac{d\vec{e_\theta}}{d\theta} = -\vec{e_r}$$

$$\boxed{\frac{d\vec{e_r}}{d\theta} = \vec{e_\theta}, \qquad \frac{d\vec{e_\theta}}{d\theta} = -\vec{e_r}}$$

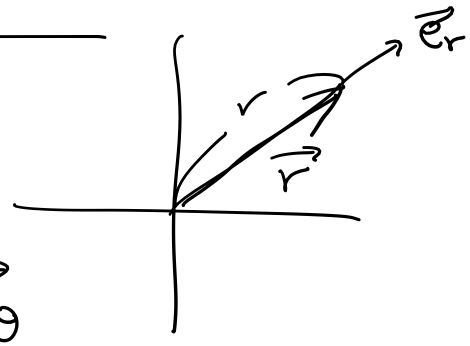
$$\underbrace{\frac{d\vec{e_r}(\theta(t))}{dt}} = \frac{d\vec{e_r}}{d\theta}\frac{d\theta}{dt} = \vec{e_\theta}\,\dot\theta, \qquad \frac{d\vec{e_\theta}}{dt} = \frac{d\vec{e_\theta}}{d\theta}\frac{d\theta}{dt} = -\vec{e_r}\,\dot\theta$$

$\vec r = r\vec{e_r}$ ((cf) $\vec r = \vec i\, x + \vec j\, y = (x, y)$)

$[(r, 0)]$

$$\vec v = \frac{d\vec r}{dt} = \dot{\vec r} = \dot r\,\vec{e_r} + r\,\dot{\vec{e_r}} = \underline{\dot r\,\vec{e_r} + r\dot\theta\,\vec{e_\theta}}$$

$$\left(= (\dot r, r\dot\theta)\right)$$



$$\vec{v} = \dot{r}\vec{e_r} + r\dot{\theta}\vec{e_\theta}$$

$$\dot{\vec{v}} = \vec{a} = \ddot{r}\vec{e_r} + \dot{r}\dot{\vec{e_r}} + \dot{r}\dot{\theta}\vec{e_\theta} + r\ddot{\theta}\vec{e_\theta} + r\dot{\theta}\dot{\vec{e_\theta}}$$

$$\dot{\vec{e_r}} = \dot{\theta}\vec{e_\theta}$$

$$\dot{\vec{e_\theta}} \to -\dot{\theta}\vec{e_r}$$

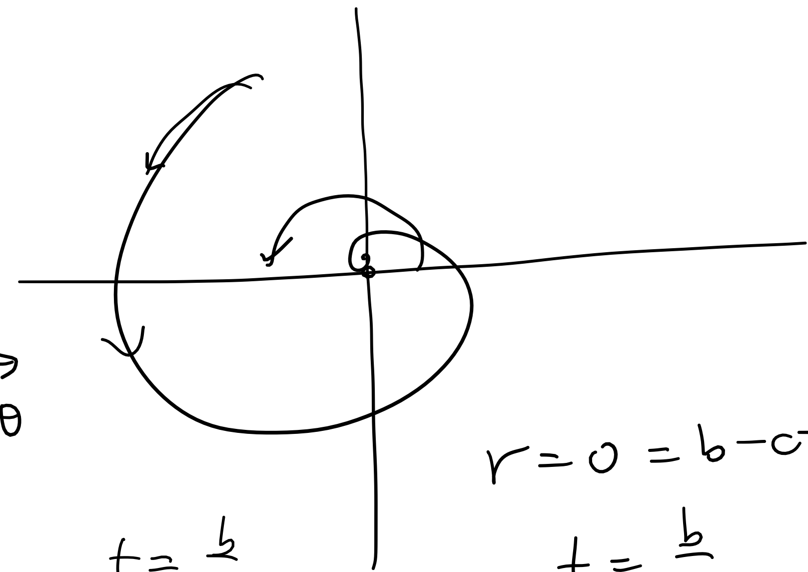
$$\therefore \vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e_r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{e_\theta}$$

$$(\vec{a})_r = \ddot{r} - r\dot{\theta}^2$$

$$(\vec{a})_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} = \frac{1}{r}\underbrace{\frac{d}{dt}(r^2\dot{\theta})}_{2r\dot{r}\dot{\theta} + r^2\ddot{\theta}}$$

(Ex)

$$\begin{cases} r(t) = b - ct \\ \dot{\theta}(t) = kt \longrightarrow \theta(t) = \frac{1}{2}kt^2 \end{cases}$$



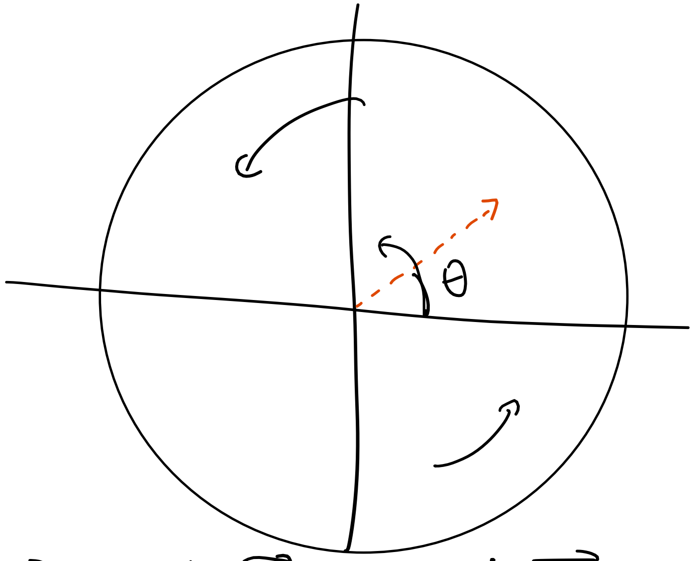
$$r = 0 = b - ct$$

$$t = \frac{b}{c}$$

$$\vec{v} = -c\vec{e_r} + (b-ct)kt\vec{e_\theta}$$

$$v = \sqrt{c^2 + (b-ct)^2k^2t^2} \xrightarrow{t = \frac{b}{c}} v = c$$

(ex)



$r = bt^2$

$\theta = \omega t$

$\vec{v} = 2bt\,\vec{e_r} + b\omega t^2\,\vec{e_\theta}$

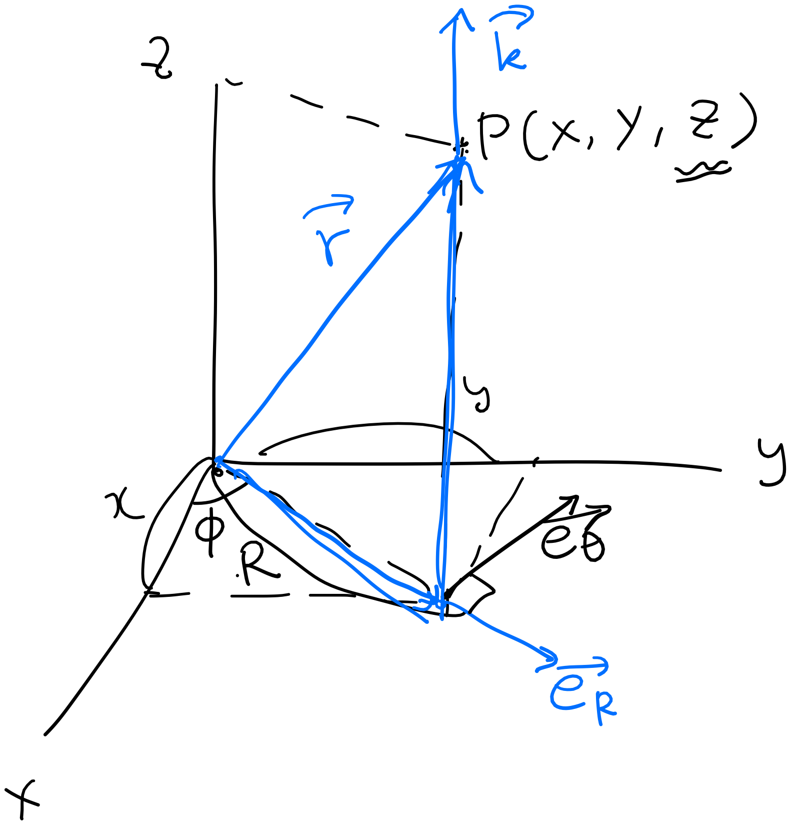
$\vec{a} = (2b - b\omega^2 t^2)\vec{e_r} + (4b\omega t)\vec{e_\theta}$

$\vec{v} = \dot{r}\vec{e_r} + r\dot{\theta}\vec{e_\theta}$

$\therefore\ \vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e_r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{e_\theta}$

1.12.

극좌표 → 3차원 좌표 $\begin{cases} \text{원통 좌표계} \\ \text{구면 〃} \end{cases}$



원통 (R, ϕ, z)

$$\vec{r} = R\vec{e_R} + z\vec{k}$$

↓

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{R}\vec{e_R} + R\dot{\phi}\vec{e_\phi} + \dot{z}\vec{k}$$

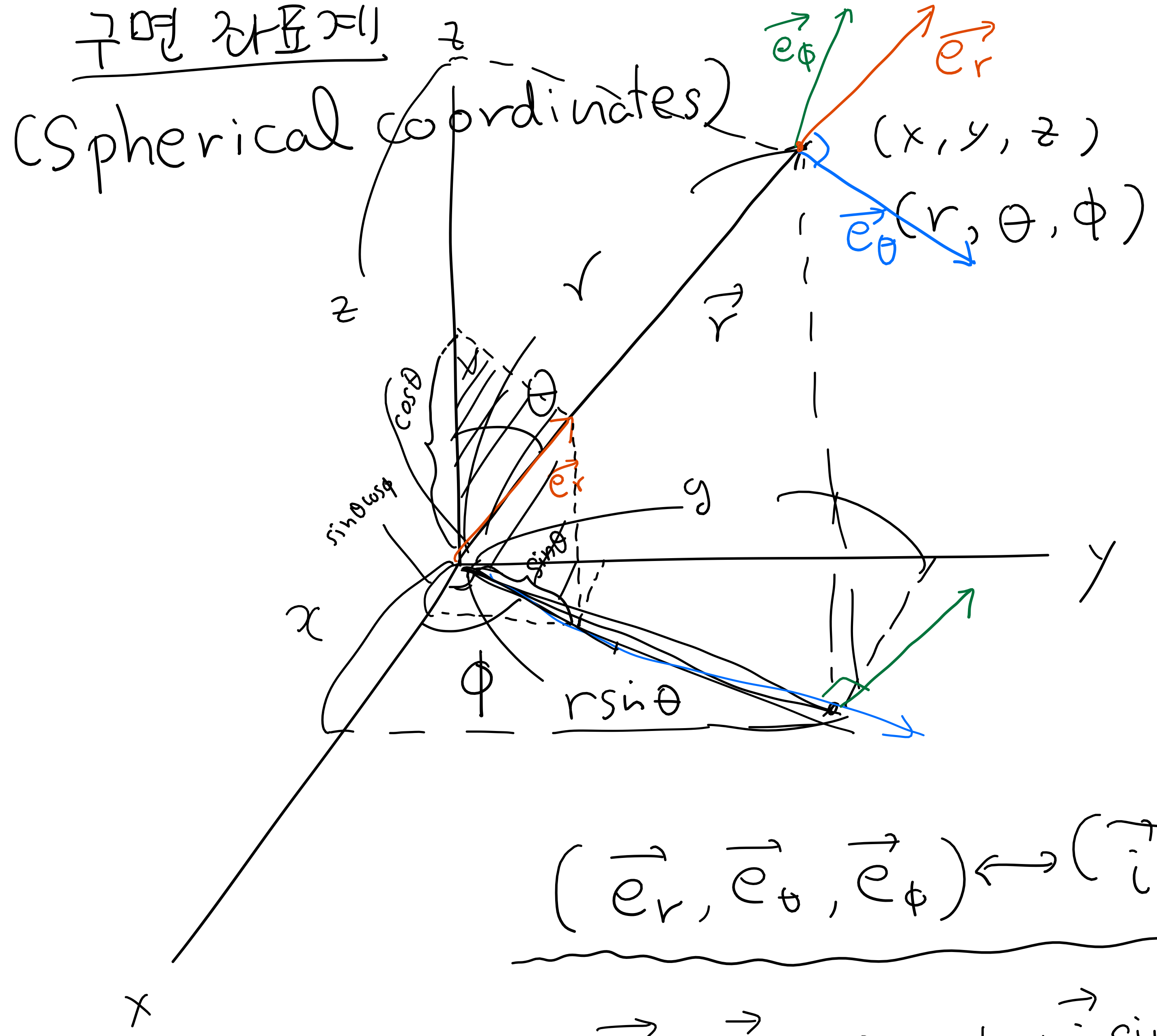
$$\vec{a} = (\ddot{R} - R\dot{\phi}^2)\vec{e_R} + (2\dot{R}\dot{\phi} + R\ddot{\phi})\vec{e_\phi} + \ddot{z}\vec{k}$$

$$\vec{e_R} = \vec{i}\cos\phi + \vec{j}\sin\phi$$

$$\vec{e_\phi} = -\vec{i}\sin\phi + \vec{j}\cos\phi$$

$$\vec{k} = \vec{k}$$

구면 좌표계 (Spherical coordinates)



$$z = r\cos\theta$$

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$r = \sqrt{x^2+y^2+z^2}$$

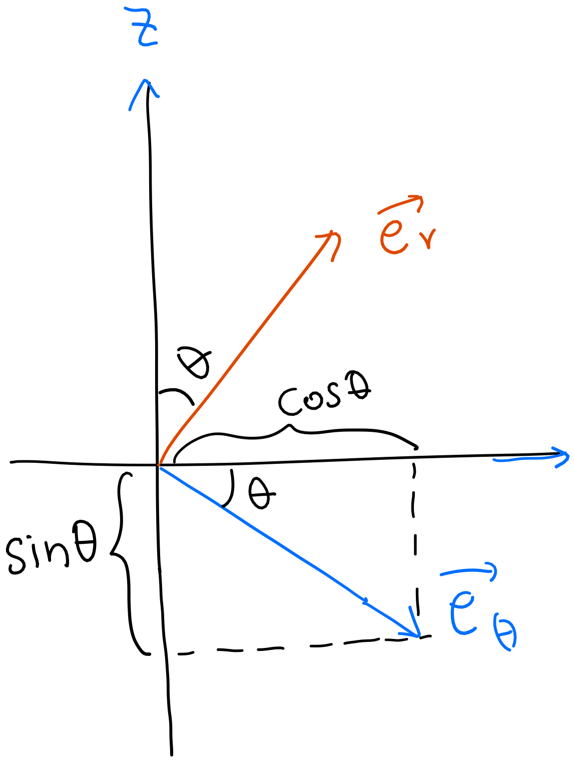
$$\theta = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}}$$

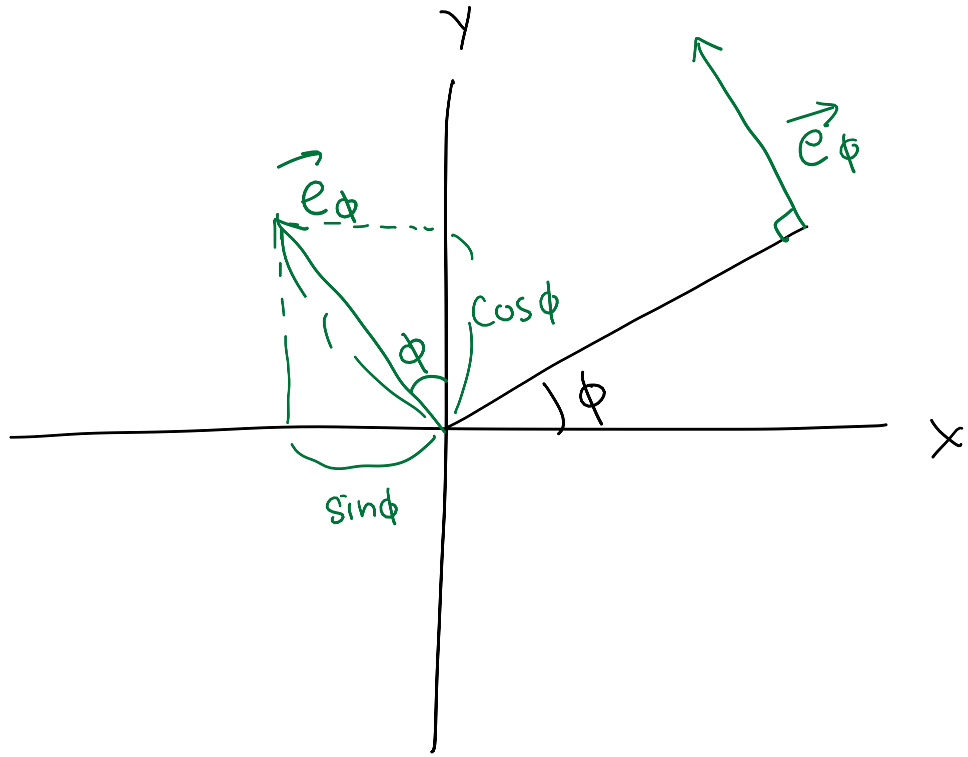
$$(\vec{e_r}, \vec{e_\theta}, \vec{e_\phi}) \longleftrightarrow (\vec{i}, \vec{j}, \vec{k})$$

$$\vec{e_r} = \vec{i}\sin\theta\cos\phi + \vec{j}\sin\theta\sin\phi + \vec{k}\cos\theta$$

$$\vec{e_\theta} = \vec{i}\cos\theta\cos\phi + \vec{j}\cos\theta\sin\phi - \vec{k}\sin\theta$$

$$\vec{e_\phi} = -\vec{i}\sin\phi + \vec{j}\cos\phi$$


z
$\vec{e}_r$
θ
$\cos\theta$
θ
$\sin\theta$
$\vec{e}_\theta$


y
$\vec{e}_\phi$
$\vec{e}_\phi$
$\cos\phi$
ϕ
ϕ
x
$\sin\phi$

$$\vec{e_r} = \vec{i}\sin\theta\cos\phi + \vec{j}\sin\theta\sin\phi + \vec{k}\cos\theta \leftarrow \times\sin\theta$$

$$\vec{e_\theta} = \vec{i}\cos\theta\cos\phi + \vec{j}\cos\theta\sin\phi - \vec{k}\sin\theta \leftarrow \times\cos\theta$$

$$\vec{e_\phi} = -\vec{i}\sin\phi + \vec{j}\cos\phi \leftarrow$$

$$\frac{d\vec{e_r}}{dt} = \vec{i}\left(\dot\theta\cos\theta\cos\phi - \dot\phi\sin\theta\sin\phi\right) + \vec{j}\left(\dot\theta\cos\theta\sin\phi + \dot\phi\sin\theta\cos\phi\right) - \vec{k}\,\dot\theta\sin\theta = \dot\theta\vec{e_\theta} + \dot\phi\sin\theta\vec{e_\phi}$$

$$\frac{d\vec{e_\theta}}{dt} = \vec{i}\left(-\dot\theta\sin\theta\cos\phi - \dot\phi\cos\theta\sin\phi\right) + \vec{j}\left(-\dot\theta\sin\theta\sin\phi + \dot\phi\cos\theta\cos\phi\right) - \vec{k}\,\dot\theta\cos\theta = -\dot\theta\vec{e_r} + \dot\phi\cos\theta\vec{e_\phi}$$

$$\frac{d\vec{e_\phi}}{dt} = -\vec{i}\,\dot\phi\cos\phi - \vec{j}\,\dot\phi\sin\phi = -\dot\phi\left(\vec{i}\cos\phi + \vec{j}\sin\phi\right)$$

$$= -\dot\phi\left(\sin\theta\,\vec{e_r} + \cos\theta\,\vec{e_\theta}\right)$$

$$\boxed{\begin{aligned} \frac{d}{dt}\vec{e_r} &= \vec{e_\theta}\,\dot{\theta} + \vec{e_\phi}\,\dot{\phi}\sin\theta \\ \frac{d}{dt}\vec{e_\theta} &= -\vec{e_\theta}\,\dot{\theta} + \vec{e_\phi}\,\dot{\phi}\cos\theta \\ \frac{d}{dt}\vec{e_\phi} &= -\vec{e_r}\,\dot{\phi}\sin\theta - \vec{e_\phi}\dot{\phi}\cos\theta \end{aligned}}$$

$$\vec{r} = r\,\vec{e_r} \longrightarrow \vec{v} = \frac{d\vec{r}}{dt} = \dot{r}\,\vec{e_r} + r\frac{d\vec{e_r}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{r}\,\vec{e_r} + 2\dot{r}\frac{d\vec{e_r}}{dt}$$

$$+ r\Big(\ddot{\theta}\,\vec{e_\theta} + \dot{\theta}\frac{d\vec{e_\theta}}{dt} + \ddot{\phi}\sin\theta\,\vec{e_\phi} + \dot{\phi}\,\dot{\theta}\cos\theta\,\vec{e_\phi} + \dot{\phi}\sin\theta\frac{d\vec{e_\phi}}{dt}\Big)$$

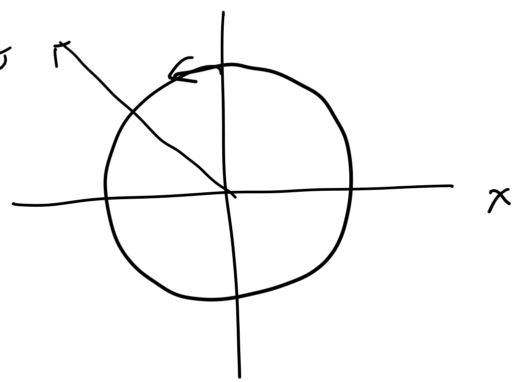
$$= (1.12.14)$$

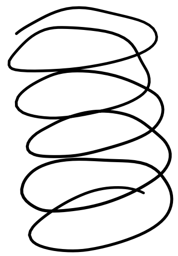
(Ex) $R = b \qquad \phi = \omega t, \quad z = ct$

$\dot{R} = 0$

$\ddot{R} = 0$

$\dot{\phi} = \omega, \quad \ddot{\phi} = 0$

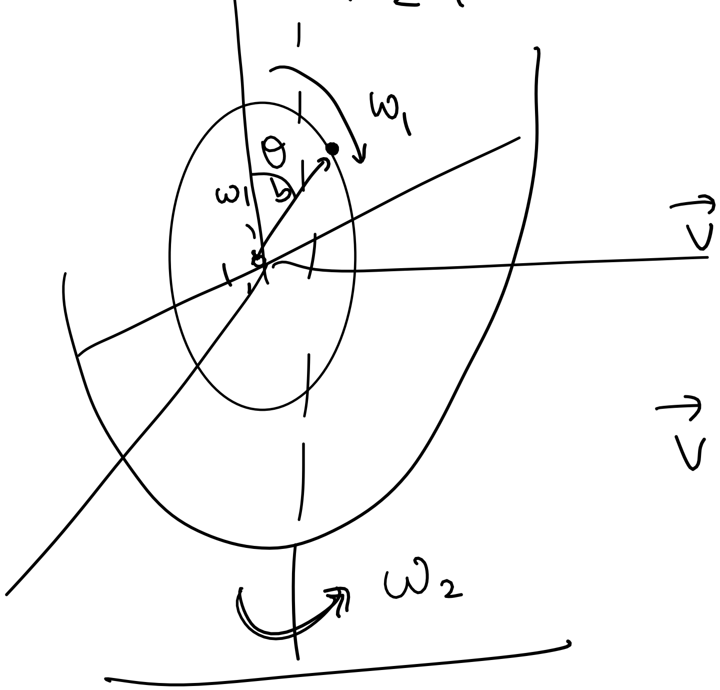




$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{R}\,\vec{e}_R + R\dot{\phi}\,\vec{e}_\phi + \dot{z}\,\vec{k} = b\omega\,\vec{e}_\phi + c\,\vec{k}$$

$$\vec{a} = (\ddot{R} - R\dot{\phi}^2)\,\vec{e}_R + (2\dot{R}\dot{\phi} + R\ddot{\phi})\,\vec{e}_\phi + \ddot{z}\,\vec{k} = -b\omega^2\,\vec{e}_R$$

(Ex)



$r = b \qquad \phi = \omega_2 t \qquad \theta = \omega_1 t$

$\dot{r} = 0 \qquad \dot{\phi} = \omega_2 \qquad \dot{\theta} = \omega_1$

$$\vec{v} = \dot{r}\,\vec{e}_r + r\left(\vec{e}_\theta\,\dot{\theta} + \vec{e}_\phi\,\dot{\phi}\sin\theta\right)$$

$$\vec{v} = b\left(\omega_1\,\vec{e}_\theta + \omega_2 \sin\omega_1 t\,\vec{e}_\phi\right)$$