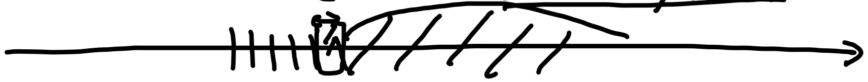
Chap 8. Critical phenomena

$\varepsilon \sim$ New Physics scale



realspace

$\Lambda_{\text{cutoff}} \sim \frac{1}{\varepsilon}$

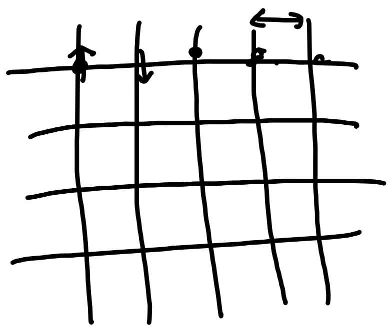
mom. space

$V(r) = \frac{k}{r} \rightarrow$ $\bar{e}$

QFT $\Longleftrightarrow$ Statistical Mechanics

Pathintegral.

continuum



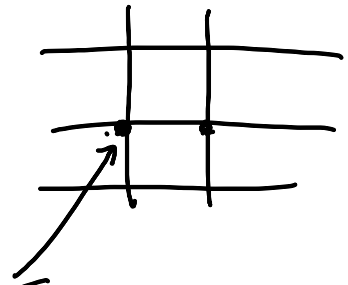
(격자가 역할, atom)

N.P.

quantum fluctuation

thermal fluctuation

$$E = -\underbrace{J}\sum_{\underbrace{\langle ij \rangle}_{n.n.}} \sigma_i \sigma_j$$



$\sigma_i = \pm 1$

$$\underline{\frac{1}{\beta} = kT \gg J} \rightarrow \beta J \ll 1$$

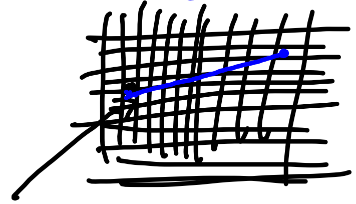
$e^{-\beta J}$ $\pm\ \pm$

$e^{\beta J}$ $\pm\ \mp$

$e^{-2\beta J} \sim 1$

magnetization

$$M = \langle \sigma \rangle = 0$$



$+ \rightarrow -$

when H is applied : $\underbrace{-\left(\sum_i \sigma_i\right) \cdot H}$

$\uparrow$

external mag. field

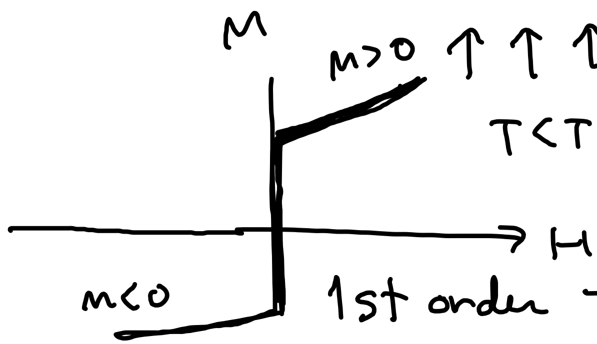
phase diagram.

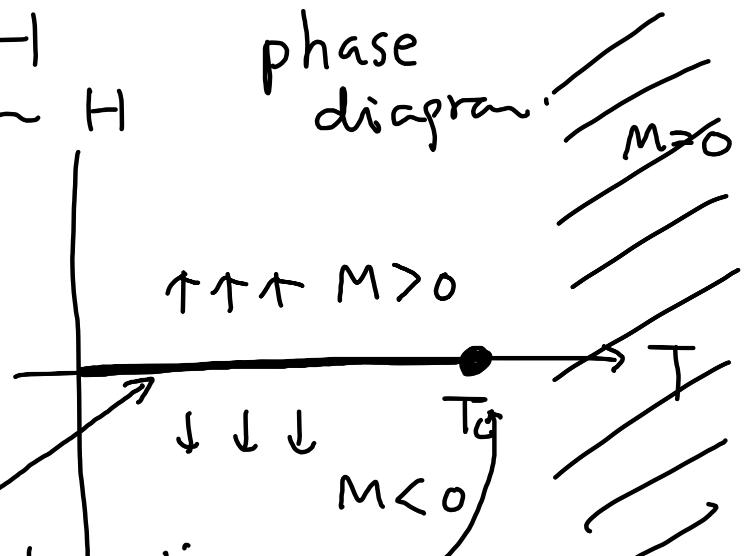
order parameter

$T < T_c$ $\uparrow\ \uparrow\ \uparrow\ \uparrow$

$\uparrow\ \uparrow\ \uparrow\ \uparrow$

$T < T_c$





phase transiti

2nd order

div. (corr. lgth)

Gibbs free energy

$$G(M) \quad : \quad \left.\frac{\partial G(M)}{\partial M}\right|_T = H$$

if $H=0 \rightarrow \frac{\partial G}{\partial M} = 0 \rightarrow G(M) = A + BM^2 + CM^4 + \cdots$

$(M \ll 1)$

T의 함수

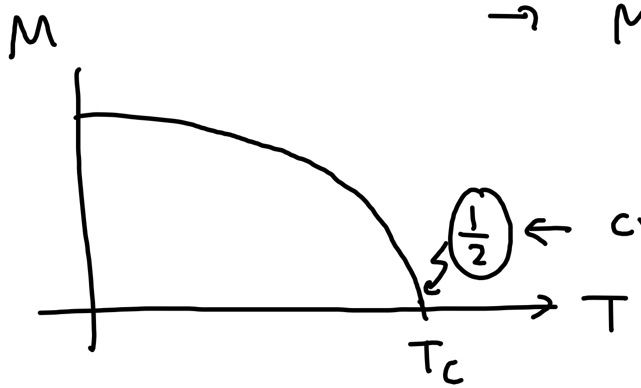
$$\left.\frac{\partial G}{\partial M}\right|_{T=T_c} = 0 \rightarrow B = \begin{cases} b(T-T_c) & T<T_c \\ 0 & T>T_c \end{cases}$$

① $H=0$

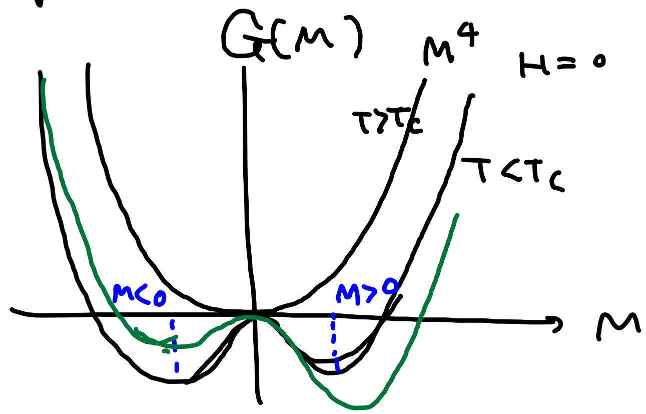
$$\frac{\partial G}{\partial M} = 0 \rightarrow 2BM + 4CM^3 = 0$$

$$\rightarrow M = \begin{cases} 0 \\ \frac{b}{2c}(T_c - T)^{\frac{1}{2}} \end{cases}$$

Landau



← critical exponent



② $H \neq 0$

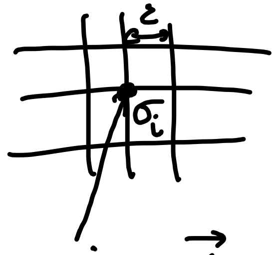
$$G = G_0 - HM$$

QFT description

$G(M)$

$\uparrow$

$\langle S(\vec{x}) \rangle$



$\varepsilon \to 0$

$\longrightarrow \sum_{\langle ij \rangle} \sigma_i \sigma_j$

$i \to \vec{x}$

$\sigma_i \to S(\vec{x})$

$\longrightarrow S(\vec{x})\, S(\vec{x}+\vec{\varepsilon})$

$\approx S(\vec{x}) \vec{\nabla}^2 S \cdot \vec{\varepsilon}^{\,2}$

$$G = \int d^3x \left[\frac{1}{2} (\vec{\nabla} S)^2 + b(T-T_c) S^2(\vec{x}) + c\, S^4(\vec{x}) - H(\vec{x}) S(\vec{x}) \right]$$

$\downarrow$ class. eq.

$$0 = \frac{\delta G}{\delta S} \to \quad -\nabla^2 S + 2b(T-T_c) S \underbrace{+ 4c S^3}_{\approx 0} - H(\vec{x}) = 0$$

$$\Big[-\nabla^2 + \underbrace{2b(T-T_c)}_{\overset{\shortparallel}{m^2 = \frac{1}{\xi^2}}} \Big] S(\vec{x}) = H(\vec{x}) = H_0\, \delta^{(3)}(\vec{x})$$

$\longrightarrow \xi = \frac{1}{\sqrt{2b(T-T_c)}} \sim (T-T_c)^{-\frac{1}{2}}$

$\uparrow$ critical exponent.

if $H(\vec{x}) = H_0\, \delta^{(3)}(\vec{x})$

F.T. $\downarrow$

$$\left(|\vec{k}|^2 + m^2 \right) \tilde{S}(\vec{k}) = H_0$$

$$\tilde{S}(\vec{k}) = \frac{H_0}{k^2 + m^2} \Rightarrow S(\vec{x}) = \int \frac{e^{i\vec{k}\cdot\vec{x}}}{k^2 + m^2} \frac{d^3k}{(2\pi)^3}$$

$\langle S(\vec{x}) S(0) \rangle \sim$ "

$$\frac{H_0}{4\pi} \frac{1}{r} e^{-r/\sqrt{2b(T-T_c)}} \longleftarrow$$

Chap 9. Functional Method

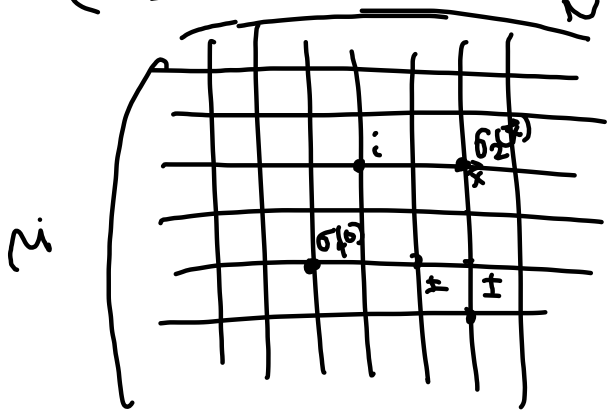
QM. $\langle \vec{x}_f, t_f | \vec{x}_i, t_i \rangle \longrightarrow \int [\mathcal{D}x]$

QFT. $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$

$\hookrightarrow H = \int d^3x \left[\frac{1}{2}\pi^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + V(\phi) \right]$

$$\langle \phi_b(\vec{x}), t_f \to \infty \,|\, \phi_a(\vec{x}), t_i \to -\infty \rangle = \int [\mathcal{D}\phi]\, e^{i\int dt \int d^3x\, \mathcal{L}[\phi, \partial_\mu \phi]}$$

(cf) Stat mech.



partition function

$-J\sum_{\langle ij \rangle} \sigma_i \sigma_j$

$$\to \quad Z = \sum_{\substack{\text{all} \\ \text{configuration}}} e^{-\beta E(\{\sigma_i\})}$$

$\{\sigma_i = + \text{ or } -\}$

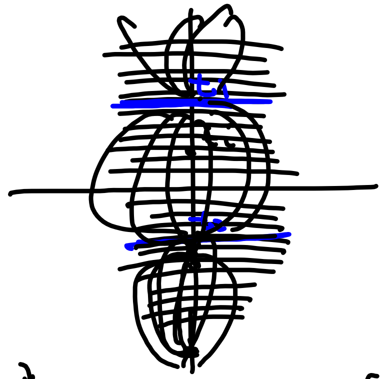
2^{MN}

correlation function

$$\langle \sigma(\vec{x})\sigma(0) \rangle = \frac{\sum_{\text{all config}} \sigma(\vec{x})\sigma(0)\, e^{-\beta E}}{Z}$$

$$\int [\mathcal{D}\phi]\ \phi(x_1)\,\phi(x_2)\, e^{iS}$$

$$\underline{\prod_i [\mathcal{D}\phi(x_i, t_i)]}\quad \phi(t_1)\quad \phi(t_2) \qquad \int_{-\infty}^{t_1}$$



$$= \int_{-\infty<t<t_2} [\mathcal{D}\phi]\, e^{iS} \cdot \int \mathcal{D}\phi(x,t_2)\ \phi(x_2)\, e^{iS} \quad \int_{t_1<t<\infty} \mathcal{D}\phi\ e^{iS}$$

$$\int \mathcal{D}\phi(x_1,t_1)\ \phi(x_1,t_1)\, e^{iS}$$

$$= \langle \phi_b |\, \underbrace{e^{-iH(-T-x_2^0)}\, \phi(x_2)\, e^{-iH(x_2^0-x_1^0)}\, \phi(x_1)\, e^{-iH(x_1^0-T)}}_{T\left\{ \phi(x_1)\,\phi(x_2)\, e^{-i\int_{-T}^{T} H\, dt} \right\}}\, |\phi_a\rangle$$

operator

$$\Rightarrow \quad \langle \Omega | T\{ \hat{\phi}(x_1)\, \hat{\phi}(x_2) \cdots \} | \Omega \rangle = \frac{\int \mathcal{D}\phi\ \phi(x_1) \cdots e^{iS}}{Z = \int \mathcal{D}\phi\ e^{iS}}$$

Feynman Rules

real $\qquad\qquad\qquad\qquad$ complex

$$\mathcal{D}\phi = \prod_i d\phi(x_i) \quad , \quad \phi(x_i) = \frac{1}{V}\sum_n e^{-i k_n \cdot x_i}\,\tilde{\phi}(k_n)$$

fourvectors

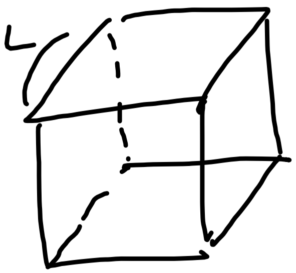
$V = L^4$

$$k_n^\mu \equiv \frac{2\pi}{L}\, n^\mu$$

$$\left(\frac{1}{V}\sum_n \overset{L\to\infty}{=} \int \frac{d^4k}{(2\pi)^4} \right)$$

$$\tilde{\phi}^*(k) = \tilde{\phi}(-k)$$

$$\tilde{\phi} = \mathrm{Re}\,\tilde{\phi} + i\,\mathrm{Im}\,\tilde{\phi}$$



$$S_0 = \int d^4x \left[\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 \right] = -\frac{1}{V}\sum_n \frac{1}{2}(m^2 - k_n^2)\,|\tilde{\phi}(k_n)|^2$$

$$|\tilde{\phi}(k_n)|^2 \to (\mathrm{Re}\,\tilde{\phi}_n)^2 + (\mathrm{Im}\,\tilde{\phi}_n)^2$$

$$\int \mathcal{D}\phi\, e^{iS_0} = \prod_n \left(\int d\,\mathrm{Re}\,\tilde{\phi}_n\, d\,\mathrm{Im}\,\tilde{\phi}_n \right) e^{-\frac{i}{V}\sum_n \frac{1}{2}(m^2 - k_n^2)\left(\quad \right)}$$

$$\prod_{n,\, k_n^0 > 0} e^{-\overbrace{\frac{i}{V}(m^2 - k_n^2)}^{a}(\mathrm{Re}\,\tilde{\phi}_n)^2} \times \prod_{n,\, k_n^0 > 0} e^{-\overbrace{\frac{i}{V}(m^2 - k_n^2)}^{a}(\mathrm{Im}\,\tilde{\phi}_n)^2}$$

$n^0 \to -n^0$

$k_n^0 \to -k_n^0$

$$\int_{-\infty}^{\infty} dx\, e^{-ax^2} = \sqrt{\frac{\pi}{a}}$$

$$= \prod_{k_n^0 > 0} \sqrt{\frac{\pi V}{i(m^2 - k_n^2)}} \sqrt{\frac{\pi V}{i(m^2 - k_n^2)}} = \prod_{\text{all } k_n} \sqrt{\frac{\pi V}{i(m^2 - k_n^2)}}$$

B ; $N \times N$ matrix

$\vec{\xi}$; N vector ; $\displaystyle\int e^{-\underbrace{\vec{\xi}^{\,T} B\, \vec{\xi}}_{\sum b_i x_i^2}}\, d\xi_1 \cdots d\xi_N$

$B \vec{x}_i = b_i \vec{x}_i$

$$= \underset{\overset{\shortparallel}{1}}{J} \int dx_1 \cdots dx_N\, e^{-\sum_i b_i x_i^2} = \prod_i \sqrt{\frac{\pi}{b_i}} = \frac{\text{const}}{\sqrt{\underbrace{\prod_i b_i}_{\det B}}}.$$

$$\boxed{\int [\mathcal{D}\xi]\, e^{-\xi^T B \xi} = \text{const} \times (\det B)^{-\frac{1}{2}}}$$

$$\to \quad S_0 = \frac{1}{2} \int d^4x \left(\underbrace{(\partial_\mu \phi)^2}_{\underbrace{\partial_\mu(\phi\, \partial^\mu \phi)}_{\text{surface} \to 0} - \phi\, \partial^2 \phi} - m^2 \phi^2 \right) = \frac{1}{2} \int d^4x\, e^{-\phi \overbrace{(\partial^2 + m^2)}^{B} \phi}$$

$$\phi(x) = \begin{pmatrix} \vdots \\ x \to \phi(x) \\ \vdots \end{pmatrix}$$

$$= \text{const} \cdot \times \left[\det(\partial^2 + m^2)\right]^{-\frac{1}{2}}$$

correlation

$$\phi(x_1)\,\phi(x_2) = \frac{1}{V}\sum_m e^{-ik_m\cdot x_1}\,\underset{\underset{\tilde{\phi}_m}{\parallel}}{\tilde{\phi}(k_m)}\;\frac{1}{V}\sum_\ell e^{-ik_\ell\cdot x_2}\,\tilde{\phi}_\ell$$

$$\Rightarrow \int \mathcal{D}\phi\; \phi(x_1)\,\phi(x_2)\; e^{iS_0}$$

$$= \frac{1}{V^2}\sum_{m,\ell} e^{-i(k_m\cdot x_1 + k_\ell\cdot x_2)} \int \prod_{\substack{n\\ k_n^0>0}} d\,\mathrm{Re}\tilde{\phi}_n\, d\,\mathrm{Im}\tilde{\phi}_n \cdot \left(\mathrm{Re}\tilde{\phi}_m + i\,\mathrm{Im}\tilde{\phi}_m\right)\left(\mathrm{Re}\tilde{\phi}_\ell + i\,\mathrm{Im}\tilde{\phi}_\ell\right)$$

$\tilde{\phi}(k_m)$

$$\times\; e^{-\frac{i}{V}\sum_{\substack{n\\ k_n^0>0}} (m^2-k_n^2)\left((\mathrm{Re}\tilde{\phi}_n)^2 + (\mathrm{Im}\tilde{\phi}_n)^2\right)}$$

$$\left(\int_{-\infty}^{\infty} dx\,dy\;\underbrace{x\,y}_{x^2}\; e^{-(x^2+y^2)} = 0\right) \Rightarrow k_m = \pm k_\ell$$

$\underline{k_m = k_\ell}$ $\quad \underbrace{\mathrm{Re}\tilde{\phi}_m\,\mathrm{Re}\tilde{\phi}_\ell}_{(\mathrm{Re}\tilde{\phi}_m)^2}$

$$\left.\begin{matrix}(\mathrm{Re}\tilde{\phi}_m)^2\\ -(\mathrm{Im}\tilde{\phi}_m)^2\end{matrix}\right\} = 0$$

$$+ \quad \Rightarrow \neq 0.$$

if $k_m = -k_\ell$

$$\tilde{\phi}(k_\ell) = \tilde{\phi}(-k_m) = \tilde{\phi}(k_m)^* = \mathrm{Re}\tilde{\phi}(k_m) - i\,\mathrm{Im}\tilde{\phi}(k_m)$$

$$\tilde{\phi}^*(k) = \tilde{\phi}(-k)$$

$$\int d\mathrm{Re}\tilde{\phi}_m\, d\mathrm{Im}\tilde{\phi}_m \left(\mathrm{Re}\tilde{\phi}_m^2 + \mathrm{Im}\tilde{\phi}_m^2\right) e^{-\underbrace{\frac{i}{V}\sum_{\substack{n\to m\\ k_n^0>0}}(m^2-k_n^2)}_{\alpha}\underbrace{\left[(\mathrm{Re}\tilde{\phi}_n)^2+(\mathrm{Im}\tilde{\phi}_n)^2\right]}_{A}}$$

$$= \int d\mathrm{Re}\tilde{\phi}_m\, d\mathrm{Im}\tilde{\phi}_m\, A\, e^{-\alpha A} = \left(-\frac{\partial}{\partial\alpha}\right)\underbrace{\int d\mathrm{Re}\tilde{\phi}_m \cdots e^{-\alpha A}}_{-\frac{\pi}{\alpha}}$$

$$\frac{-i\pi V}{m^2-k_m^2} = \frac{-\pi}{\boxed{-i\left(\frac{m^2-k_m^2}{V}\right)}} \equiv \alpha$$

$$= -\frac{\pi}{\alpha^2} = \left(-\frac{\pi}{\alpha}\right)\cdot\frac{1}{\alpha}$$

$$\therefore \int \mathcal{D}\phi\, \phi(x_1)\phi(x_2)\, e^{iS_0}$$

$$= \frac{1}{V^2}\sum_m e^{-ik_m\cdot(x_1-x_2)}\left[\prod_{\substack{n\\ k_n^0>0}}\left(\frac{-i\pi V}{m^2-k_n^2}\right)\right]\cdot\frac{-iV}{m^2-k_m^2}$$

$$\int \mathcal{D}\phi\, e^{iS_0}$$

$$\langle 0|T\{\phi(x_1)\phi(x_2)\}|0\rangle = \int \frac{d^4k}{(2\pi)^4}\underbrace{\boxed{\frac{i}{k^2-m^2+i\varepsilon}}}_{\tilde{D}_F(k)} e^{-ik\cdot(x_1-x_2)} = D_F(x_1-x_2)$$

higher correlation funct.

$$\int [D\phi]\ \tilde{\phi}_m\ \tilde{\phi}_l\ \tilde{\phi}_p\ \tilde{\phi}_q \cdots e^{iS_0}$$

$S_0 \leftarrow \sum_n (\mathrm{Re}\,\tilde{\phi}_n)^2 + (\mathrm{Im}\,\tilde{\phi}_n)^2$

$\tilde{\phi}_m = \tilde{\phi}(k_m)$

$k_l = -k_m$

$k_p = -k_q$

$$\langle 0 | T\{\phi_1 \phi_2 \phi_3 \phi_4\} | 0 \rangle = D_F(1-2)\, D_F(x_3 - x_4) + \cdots$$

Vertex

$$\mathcal{L} = \mathcal{L}_0 - \frac{\lambda}{4!}\phi^4 \quad \rightarrow \quad e^{iS} = e^{iS_0 - \underbrace{\frac{i\lambda}{4!}\int \phi^4 d^4x}}$$

$\mathcal{L}_0 = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2$

$$= e^{iS_0}\left[1 - \frac{i\lambda}{4!}\int \phi^4 d^4x + \cdots\right]$$

$$\rightarrow \quad \int [D\phi]\ e^{iS}\left[1 - \frac{i\lambda}{4!}\int d^4x_1\, \phi^4(x_1) + \cdots\right]$$

Generating function

$$Z = \sum_{\text{all}} e^{-\beta \sum_{\langle i,j\rangle} \sigma_i \sigma_j}$$

$$\langle \sigma_k \sigma_0 \rangle = \frac{\sum_{\text{all}} \sigma_k \sigma_0 e^{-\beta \sum_{\langle i,j\rangle} \sigma_i \sigma_j}}{Z}$$

$$Z[J] = \sum_{\text{all}} e^{-\left(\beta \sum \sigma_i \sigma_j + \underbrace{\sum_i J_i \sigma_i}\right)}$$

$$= \frac{1}{Z} \frac{\partial^2}{\partial J_k \partial J_0} Z[J]\Big|_{J=0}$$

$$Z[0] = Z \qquad -\frac{\partial Z}{\partial J_i} = \sum_{\text{all}} \sigma_i e^{-(\dots)}$$

$$\mathcal{L}[J] = \mathcal{L}_0 + J(x)\phi(x) \to e^{i\left(S_0 + \int J(x)\phi(x)\, d^4x\right)}$$

$$Z[J] \equiv \int [\mathcal{D}\phi]\, e^{+iS_0 + i\int \underline{J(x)\phi(x)}\, d^4x}$$

$$\frac{\delta}{\delta J(y)} \int \underline{J(x)}\phi(x)\, d^4x$$

$$= \int \delta^{(4)}(x-y)\phi(x)\, d^4x$$

$$= \phi(y)$$

$$(-i)\frac{\delta Z[J]}{\delta J(y)} = \int [\mathcal{D}\phi]\, e^{iS_0 + i\int J\phi} \cdot$$

$$= \int [\mathcal{D}\phi]\, \underline{\underline{\phi(y)}}\, e^{iS_0 + i\int J\phi}$$

$$\langle 0| T\{\phi(x_1)\phi(x_2)\}|0\rangle = \frac{1}{Z_0}\left(-i\frac{\delta}{\delta J(x_1)}\right)\left(-i\frac{\delta}{\delta J(x_2)}\right) Z[J]\Big|_{J=0}$$

$$Z_0 = Z[J=0]$$

$$Z[J] = \int \mathcal{D}\phi \; e^{i\int d^4x \left(\underbrace{\frac{1}{2}(\partial_\mu\phi)^2}_{\substack{= \\ -\frac{1}{2}\phi\partial^2\phi}} - \frac{1}{2}m^2\phi^2 + J\phi\right)} \to -\frac{1}{2}\phi'(\partial^2+m^2)\phi' + \frac{1}{2}J(\partial^2+m^2)^{-1}J$$

$$-\frac{1}{2}\phi(\underbrace{\partial^2+m^2}_{A})\phi = -\frac{1}{2}\vec{\phi}^T A\vec{\phi} + \vec{J}\cdot\vec{\phi}$$

$\phi(x) \to$ index

$$\vec{\phi} = \vec{\phi}' + \underbrace{A^{-1}\cdot\vec{J}}_{\phi \text{ indepede}} \to -\frac{1}{2}(\vec{\phi}'^T + \vec{J}^T(A^{-1})^T)A(\vec{\phi}' + A^{-1}\vec{J})$$

$$(A^{-1})^T A = A^{-1}$$

$$= -\frac{1}{2}\vec{\phi}'^T A\vec{\phi}' \underbrace{-\frac{1}{2}\vec{J}\cdot\vec{\phi}' - \frac{1}{2}\vec{\phi}'\cdot\vec{J}}_{= -\cancel{\vec{\phi}'\cdot\vec{J}}}$$

$$-\frac{1}{2}\vec{J}^T A^{-1}\vec{J}$$

$$\int\mathcal{D}\phi = \int\mathcal{D}\phi'$$

$$+\vec{J}\cdot(\vec{\phi}' + A^{-1}\vec{J}) = \cancel{\vec{J}\cdot\vec{\phi}'} + \vec{J}^T A^{-1}\vec{J}$$

$$= \underbrace{-\frac{1}{2}\vec{\phi}'^T A\vec{\phi}' + \frac{1}{2}\vec{J}^T A^{-1}\vec{J}}$$

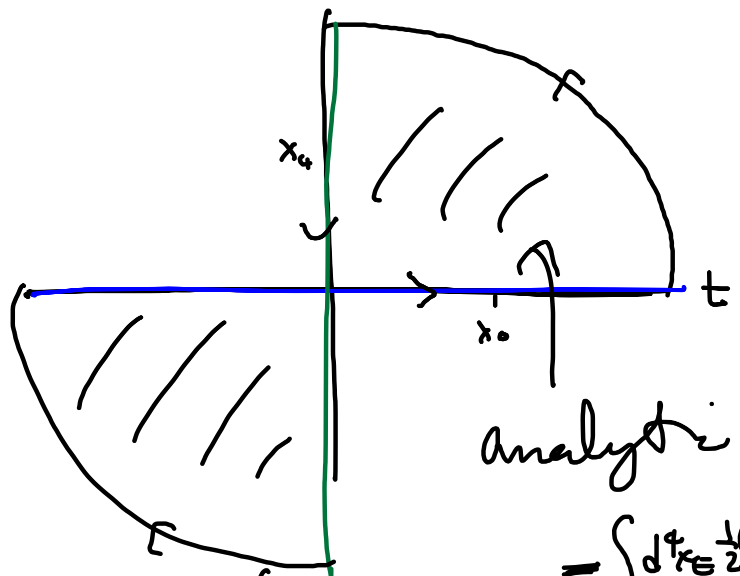
$$\therefore \quad \boxed{Z[J] = e^{-\frac{1}{2}\int d^4x\, d^4y\, J(y)\, D_F(x-y)\, J(x)}\, Z[0]}$$

$$(\partial^2 + m^2)_x \underbrace{(\partial^2 + m^2)^{-1}_y}_{(-i)D_F(x-y)} = \delta^{(4)}(x-y)$$

$$\boxed{\langle 0 | T\{\phi(x_1) \cdots \phi(x_n)\} | 0 \rangle = \frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)}\, e^{-\frac{1}{2}\int d^4x\, d^4y\, J(y)\, D_F(x-y)\, J(x)}}$$

Wick rotation

$t = X^0 = \text{real} \longrightarrow \text{imaginary} \quad x^0 = -i X^4$



$d^4x = dx^0 \, d^3\vec{x} = -i dx^4 d^3\vec{x} = -i \, d^4\underset{\uparrow \text{Euclidean}}{x}$

Minkowski $\longrightarrow$ (x^1, x^2, x^3, x^4)

$S = \int d^4x \, \mathcal{L} = -i \int d^4x_E \cdots \to e^{iS} = e^{+\int d^4x_E \mathcal{L}} = e^{-\int d^4x_E \frac{1}{2}(\cdots)}$

$\mathcal{L} \uparrow$

$\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 = \frac{1}{2}(\partial_0\phi)^2 - \frac{1}{2}(\vec{\nabla}\phi)^2 - \frac{1}{2}m^2\phi^2 = \underbrace{-\frac{1}{2}\left(\overbrace{(\partial_E\phi)^2 + m^2\phi^2}\right)}_{\geq 0}$

$\frac{1}{2}(\partial_0\phi)^2 = \underbrace{-\frac{1}{2}(\partial_4\phi)^2 - \frac{1}{2}(\vec{\nabla}\phi)^2}_{-\frac{1}{2}\underbrace{(\partial_E\phi)^2}_{\geq 0}}$

$(\partial_\mu\phi)^2 \not\geq 0$

$= (\partial_0\phi)^2 - (\vec{\nabla}\phi)^2$

$Z = \int [\mathcal{D}\phi_E] \, e^{-\underbrace{\int d^4x \, \mathcal{L}_E}_{= S_E}} \quad \mathcal{L}_E \geq 0$

$e^{-\beta \sum_{\langle i,j \rangle} \sigma_i \sigma_j} \quad \sum \to \int d^3\vec{x}$

$\int_{-\infty}^{\infty} dx_4 \to 2\int_0^{\infty} dx_4$ ← imaginary time

$2\int_0^{\beta = \frac{1}{T}} dx_4$

$$Z[J] = \int \mathcal{D}\phi \; e^{-\int d^4x_E \, (\mathcal{L}_E - J\phi)}$$

$$= e^{-\frac{1}{2}\int d^4x \, d^4y \; J(y) D_F(\cancel{x}-y) J(x)} \cdot Z_0.$$

9.4. photon propagator. $\langle 0 | T(A_\mu(x) A_\nu(y)) | 0 \rangle$

$$= \int \frac{d^4k}{(2\pi)^4} \boxed{\frac{i\, g_{\mu\nu}}{k^2 + i\varepsilon}} \; e^{-ik\cdot(x-y)}$$

$\underline{\partial_\mu A^\mu = 0}$

$$S = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) = \frac{1}{2} \int d^4x \; A_\mu(x) \left(\partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu \right) A_\nu(x)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \tilde{A}_\mu(k) \left(-k^2 g^{\mu\nu} + k^\mu k^\nu \right) \tilde{A}_\nu(k)$$

$$\int [\mathcal{D}A_\mu]\, e^{iS} = \det\left(\frac{\delta G}{\delta\alpha}\right)\int \mathcal{D}\alpha \int \mathcal{D}A^\alpha\, \delta(G(A^\alpha))\, e^{iS}$$

integral over all A_μ inequivalent

$$A_\mu \equiv A^\alpha_\mu = A_\mu + \frac{1}{e}\partial_\mu\alpha$$

gauge condition $G(A) = 0$ $\left((\text{ex})\ \vec{\nabla}\cdot\vec{A} = 0\right)$

$$1 = \int \underbrace{\mathcal{D}G}_{\substack{\shortparallel \\ \underbrace{\det\left(\frac{\delta G}{\delta\alpha}\right)}_{\text{Jacobian}}\mathcal{D}\alpha}} \cdot\ \delta\left(G(A^\alpha)\right)$$

$$G(A) \equiv \partial^\mu A_\mu - \omega$$

$$1 = N(\xi)\int \mathcal{D}\omega\, e^{-i\int d^4x \frac{\omega^2}{2\xi}}$$

$$\int \mathcal{D}A\, e^{iS} = \int \mathcal{D}\omega\, \underset{\uparrow}{J_{ac.}} \int \mathcal{D}\alpha \int \mathcal{D}A\, \underbrace{\delta(\partial^\mu A_\mu - \omega)}\, e^{iS} \cdot \left[N(\xi) \qquad e^{-i\int d^4x \frac{\omega^2}{\xi}} \right]$$

$$G(A^\alpha) = \partial^\mu \underbrace{A^\alpha_\mu}_{A_\mu + \frac{i}{e}\partial_\mu \alpha} - \omega \qquad \det \frac{\delta G}{\delta \alpha} = \det\left(\frac{i}{e}\partial^2\right)$$

$$= J_{ac.} \int \mathcal{D}\alpha \int \mathcal{D}A\; e^{-i\int d^4x \frac{1}{2\xi}(\partial^\mu A_\mu)^2 + iS}$$

$$\frac{1}{2}\int d^4x\; \underbrace{A_\mu(x)}\, \underbrace{\left(\partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu\right)}\, A_\nu(x)$$

$$+ \frac{1}{\xi}\partial^\mu\partial^\nu \hookrightarrow \xi k^\mu k^\nu$$

$$\Rightarrow \qquad \frac{-i\, g^{\mu\nu}}{k^2 + i\varepsilon} \rightarrow \frac{-i}{k^2 + i\varepsilon}\left(g^{\mu\nu} - (1-\xi)\frac{k^\mu k^\nu}{k^2} \right)$$

$\xi = 1 \rightarrow$ Feynman gauge

$\xi = 0 \rightarrow$ Landau "

9.5. Spinor

$$\{\hat{\psi}(x), \hat{\psi}^{\dagger}(y)\} = \delta^{(3)}(x-y) \longrightarrow \int \mathcal{D}\phi$$

$$\rightarrow \int \mathcal{D}\psi$$

~~complex~~

Grassmann number

$1 \cdot 2 = 2 \cdot 1$

$$\eta\theta = -\theta\eta \longrightarrow \theta_1 \cdot \theta_2 = -\theta_2 \cdot \theta_1$$

$$\theta \cdot \theta = -\theta \cdot \theta = 0$$

$$f(\theta) = \underbrace{f(0) + f'(0)\theta}_{} + \underbrace{f''(0)\theta^2 + \cdots}_{0}$$

$$\frac{df}{d\theta} = f'(0) \quad ; \quad \int d\theta\, f(\theta) = \int d\theta \left(f(0) + f'(0)\,\theta \right)$$

$d(x+a) = dx$

$$= \int \underbrace{d(\theta+\eta)}_{= d\theta} \left(\underbrace{f(0) + f'(0)(\theta+\eta)}_{[f(0)+f'(0)\eta] + f'(0)\theta} \right)$$

$$\Rightarrow \int d\theta \left(f(0) + f'(0)\,\theta \right) = f'(0)$$

$$\int d\theta \cdot \theta = 1 \Leftarrow \int d\theta\, f(\theta) = \frac{df}{d\theta} = f'(0)$$

complex Grassmann $\begin{pmatrix}\theta \\ \bar\theta\end{pmatrix} = \dfrac{\theta_1 \pm i\theta_2}{\sqrt{2}} \rightarrow \theta\bar\theta \neq 0 \quad (\theta\bar\theta)^2 = 0$

$$\theta^2 = \underset{0}{\theta_1^2} - \underset{0}{\theta_2^2} + i\underbrace{(\theta_1\theta_2 + \theta_2\theta_1)}_{0}$$

$$\int d\bar\theta\, d\theta\, e^{-\bar\theta b \theta} = \int d\bar\theta\, d\theta \Big(1 - \bar\theta b \theta + \underbrace{\tfrac{1}{2}(\bar\theta b \theta)^2 + \cdots}_{0} \Big)$$

$$= -\int d\bar\theta\, d\theta\ \bar\theta b \theta$$

$$= \int d\theta \underbrace{\int d\bar\theta\ \bar\theta}_{1}\, b\,\theta = b \underbrace{\int d\theta\ \theta}_{1} = b$$

$$\int \prod_i d\bar\theta_i\, d\theta_i\ e^{-\vec{\bar\theta}^T B \vec\theta} = \int \prod_i d\bar\theta_i'\, d\theta_i'\ e^{-\sum_i \bar\theta_i' b_i \theta_i'}$$

$$\text{"}\ \theta_k \bar\theta_l = \underline{\det B \cdot (B^{-1})_{kl}} \qquad = b_1 b_2 \cdots b_N = \det B$$

Fermionic generating function

$$Z[\bar{\eta}, \eta] = \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi \, e^{i\int d^4x \left[\bar{\Psi}(i\not{\partial} - m)\Psi + \bar{\eta}\Psi + \eta\bar{\Psi} \right]}$$

$$= e^{-\int d^4x\, d^4y\, \bar{\eta}(x) \underbrace{(i\not{\partial} - m)^{-1}\delta(x-y)}_{S_F(x-y)} \eta(y)} Z[0]$$

$$\langle 0 | T\{ \Psi(x_1) \bar{\Psi}(x_2) \} | 0 \rangle = -i \frac{\delta}{\delta \bar{\eta}(x_1)} \frac{\delta}{\delta \eta(x_2)} e^{-\int d^4x\, d^4y\, \bar{\eta}(x) S_F(x-y) \eta(y)}$$