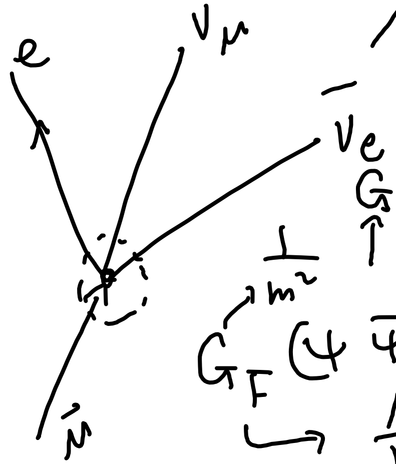
Chap4. Feynman Diagrams



GWS

$\uparrow$

$$\frac{1}{m^2}$$

$$G_F(\psi\bar{\psi})^2$$

$$\hookrightarrow \frac{\Lambda^2}{m^2}$$

$D=2$ $[\psi]=\frac{1}{2}$ $[\bar{\psi}\psi]=1$

$$\mathcal{L}=\bar{\psi}(i\not\partial-m)+\overset{[g]=0}{g}(\bar{\psi}\psi)^2$$

D

$$\overset{D}{\overset{\shortparallel}{[\mathcal{L}]}}=\bar{\psi}(i\overset{1}{\not\partial}-\overset{1}{m})\psi+\cdots$$

$$[\psi]=\frac{D-1}{2}$$

$$[\bar{\psi}\psi]=D-1$$

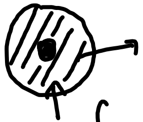
$$f(\bar{\psi}\psi)\Rightarrow\overset{[c]=2-D}{c}(\bar{\psi}\psi)^2$$

$$\overset{\shortparallel}{2D-2}>D$$

$$g(\underbrace{\bar{\psi}\partial_\mu\psi}_{D})$$

QFT with interactions → 매우 제한됨.

↳ 해를 찾기 힘듦.



→ perturbation theory

free theory (chap 2,3)

(cf) Q.M.

$$H=\frac{\vec{p}^{\,2}}{2m}+V(\vec{x})$$

아무 함수 모두 가능

- Lorentz invariant $f(\bar{\psi}\psi)$, $g(\phi)$
- Renormalizability

$\Updownarrow$

parameter의 차원이 음수는 안됨. naive dim.

$$V(\phi)=\sum_{n=0}^{\infty}V_n\phi^n$$

$$[\phi^n]=n\cdot\frac{D-2}{2}$$

$$[V_n]=D-n\frac{D-2}{2}=\frac{D(1-n)}{2}+n$$

$\downarrow$

$D=4\to 4-n\geqq 0$

$\downarrow$

$\boxed{V=\cdots+\phi^4}$ $n\leqq 4$

$D=2\to[\phi]=0\to V(\phi)m^2$

$[m]=1>0$

$\uparrow$

$$[\phi]=\frac{D-2}{2}\leftarrow\mathcal{L}=\frac{1}{2}(\partial_\mu\phi)^2-\overbrace{\frac{1}{2}m^2\phi^2}^{V(\phi)}$$

$$e^2=\frac{1}{137}$$

$[S]=0$ $\left.\begin{matrix}(\hbar\equiv1)\\ c\equiv1\end{matrix}\right\}$ Natural unit

$$e^{i\frac{S}{\hbar}}\quad\int \overset{\to -D}{d^Dx}\,\overset{\to D}{\mathcal{L}}$$

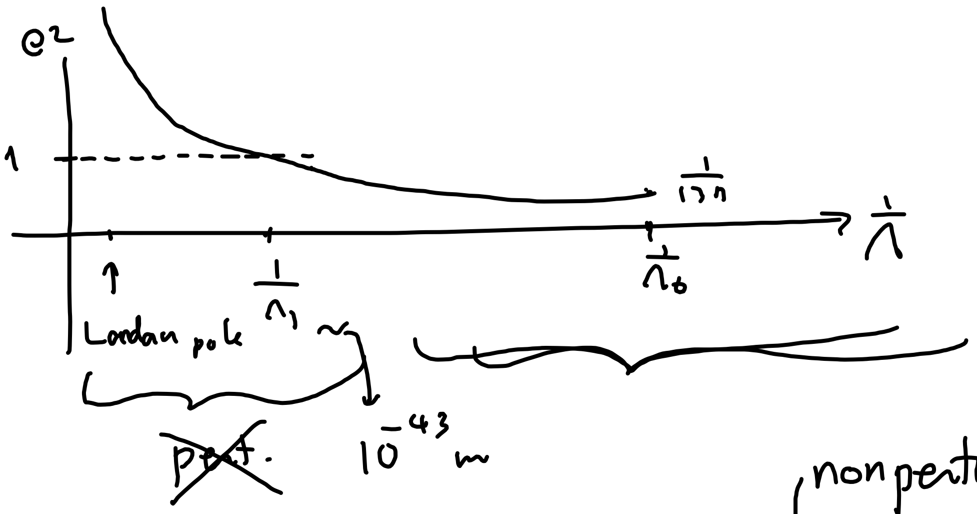
$[t]=[x]=-1$, $[\partial_\mu]=1$

$$\boxed{[\mathcal{L}]=D}$$

QED

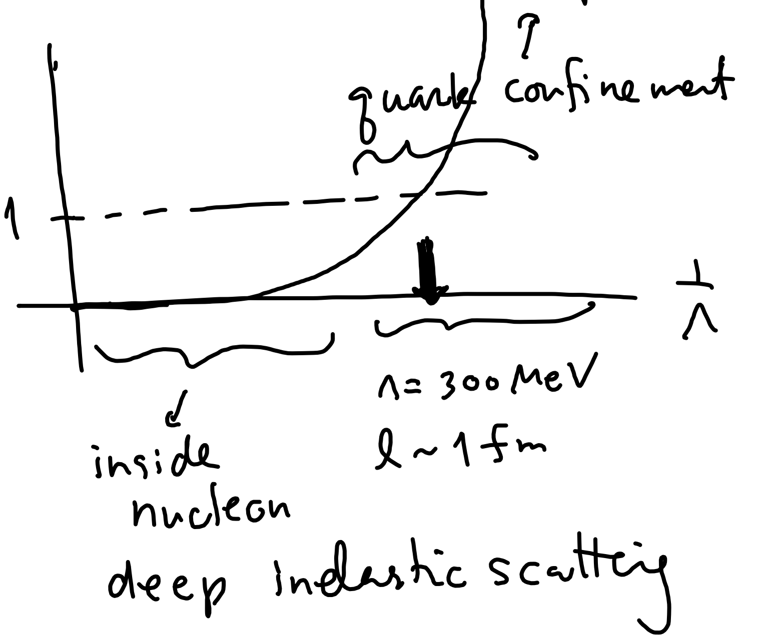
$$e^2(\Lambda) = \frac{e^2(\Lambda_0)}{1 - \underbrace{e^2 \ln\frac{\Lambda}{\Lambda_1}}_{\overset{\shortparallel}{1}}}$$

$\uparrow$



QCD

$$g^2(\Lambda) = \frac{g^2(\Lambda_0)}{1 + g^2 \ln\frac{\Lambda}{\Lambda_1}}$$



$$H = H_0 + H_{int}(t) \rightarrow [H_0, H_{int}] \neq 0$$

$$\mathcal{L}(\phi) = \mathcal{L}_0 - \underline{\mathcal{L}_{int}} \longrightarrow \mathcal{H} = H_0 + \overbrace{\mathcal{L}_{int}}^{\rightarrow H_{int}}$$

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}$$

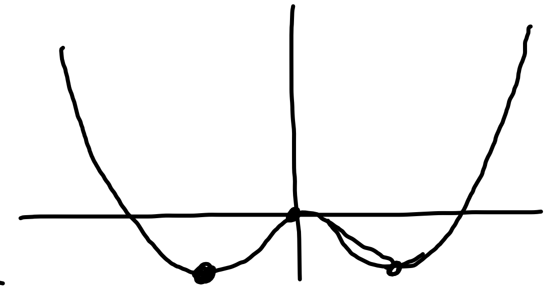
① $$\boxed{\mathcal{L}(\phi) = \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2}_{\mathcal{L}_0 \uparrow} - \frac{\lambda}{4!}\phi^4} \rightarrow \text{"}\phi^4\text{"-theory}$$

$(m^2 > 0)$

$\lambda \ll 1$ $([\lambda] = 0)$

$m^2 < 0 \rightarrow$ Higgs potential

$$V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!}\phi^4$$



② Yukawa theory

$$\boxed{\mathcal{L} = \mathcal{L}_{\text{free Dirac}} + \mathcal{L}_{\text{free scala}} - \underbrace{\overset{[g]=0}{g}}_{1} \phi \underbrace{\bar{\psi}\psi}_{3}}$$

③ QED (Quantum Electrodynamics) : $A_\mu = (\phi, \vec{A}) \rightarrow \vec{E}, \vec{B}$

$$[\Pi_{A_\mu}(x), A_\mu(y)] \sim \delta^4(x-y) \quad \text{(cf)}\ H = \frac{-\hbar^2}{2m}\left(\vec{\nabla} - \frac{e}{c}\vec{A}\right)^2 + e\phi$$

고전

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & 0 & B_3 & -B_2 \\ \vdots & & 0 & B_1 \\ & & & 0 \end{pmatrix} \rightarrow -\frac{1}{4}(F_{\mu\nu})^2 \sim (\partial_\mu A_\nu)^2$$

$\text{(cf)}\ (\partial_\mu \phi)^2$

A_μ $\quad \mu = 0,1,2,3$ → not all indep. → only two indep.

$\downarrow$

$$F_{\mu\nu}(\vec{E}, \vec{B}) = \partial_\mu \underset{\substack{\uparrow \\ \cancel{\partial_\nu \chi}}}{A_\nu} - \partial_\nu \underset{\substack{\uparrow \\ \cancel{\partial_\mu \chi}}}{A_\mu}$$

⇒ Quantization needs <u>Path integral</u>

$\nearrow$

$$A'_\mu = A_\mu + \underline{\partial_\mu \chi}$$

gauge

$c \equiv 1$

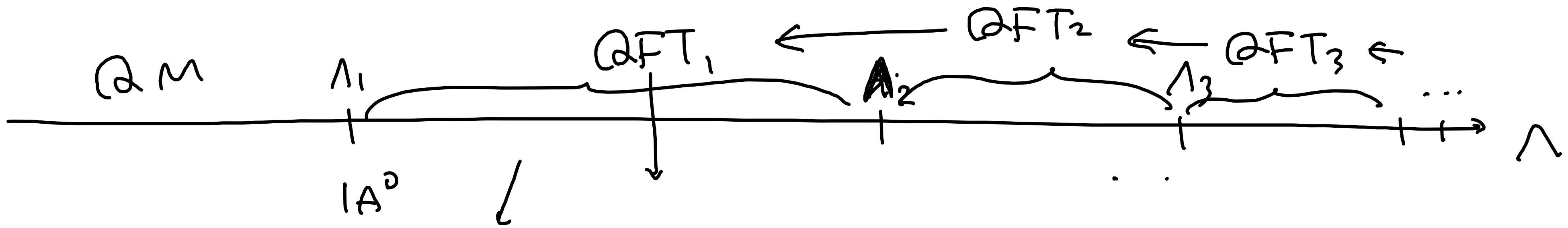
$$\boxed{\mathcal{L}_{QED} = \bar{\psi}(i\cancel{D} - m)\psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}}$$

$D_\mu = \partial_\mu + \underline{ieA_\mu}$

"covariat" derivative

$$= \underbrace{\bar{\psi}(i\cancel{\partial} - m)\psi - \frac{1}{4}F_{\mu\nu}^2}_{\mathcal{L}_0} \underbrace{- e\bar{\psi}\cancel{A}\psi}_{\mathcal{L}_{int}}$$

$\boxed{\text{Ken. Wilson}}$



$$\int_{\Lambda_1}^{\Lambda_2} dE$$

<u>effective</u> QFT.

$\xleftarrow{2L}$ a(q)

$\xleftarrow{2\pi r}$ lg

4.2. Perturbation Expansion of "Correlation function"

"ϕ^4"

$$\langle \Omega | T(\phi(x_1)\phi(x_2)\,\psi \;\cdots\;) \underbrace{|\Omega\rangle}$$

→ H_0의 ground state

$H = H_0 + H_{int} \to |\Omega\rangle \neq |0\rangle$

free

$$\langle 0 | T(\phi(x)\phi(y)) | 0 \rangle = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\varepsilon} e^{-ip\cdot(x-y)}$$

$$H = \underbrace{H_0} + \underline{\frac{\lambda}{4!}\int \phi^4 \, d^3x}$$

$$e^{iHt}\phi(\vec{x})\,e^{-iHt} = \phi(\vec{x},t) = \cancel{\phi_H(x)} =$$

$$\phi(\vec{x}) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}\left(a_{\vec{p}}\,e^{i\vec{p}\cdot\vec{x}} + a_{\vec{p}}^{\dagger}\,e^{-i\vec{p}\cdot\vec{x}}\right)$$

Interaction picture

$$\phi_I(x) = e^{iH_0t}\,\phi_S(\vec{x})\,e^{-iH_0t} = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}\left(a_{\vec{p}}\,e^{-ip\cdot x} + a_{\vec{p}}^{\dagger}\,e^{ip\cdot x}\right)$$

$p^2 = m^2$

$p^0 = E^0_{\vec{p}}$

$$e^{iH_0t}a_p e^{-iH_0t} = e^{-iE^0_{\vec{p}}t}\,a_{\vec{p}}$$

$E^0_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$

$$\phi(t,\vec{x}) = \underbrace{e^{iH(t-t_0)}\, e^{-iH_0(t-t_0)}}_{U^\dagger(t,t_0)}\; \underbrace{e^{iH_0(t-t_0)}\, \underset{S}{\phi}(t_0,\vec{x})\, e^{-iH_0(t-t_0)}}_{\phi_I}\; \underbrace{e^{iH_0(t-t_0)}\, e^{-iH(t-t_0)}}_{\equiv\, U(t,t_0)}$$

$$e^{-iH_0(t-t_0)}\, e^{iH_0(t-t_0)} = 1 \qquad e^{-iH_0(t-t_0)}\, e^{iH_0(t-t_0)} = 1$$

$$\phi(x) = U^\dagger(t,t_0)\, \phi_I\, \underline{\underline{U(t,t_0)}}$$

$$i\frac{\partial U}{\partial t} = i\left(e^{iH_0(t-t_0)}\, iH_0\, e^{-iH(t-t_0)} - e^{iH_0(t-t_0)}\, iH\, e^{-iH(t-t_0)}\right)$$

$$= \underbrace{e^{iH_0(t-t_0)} \underbrace{(H-H_0)}_{\equiv\, H_{int}} e^{-iH_0(t-t_0)}}_{(H_{int})_I}\; \underbrace{e^{iH_0(t-t_0)}\, e^{-iH(t-t_0)}}_{\equiv\, U}$$

$$\boxed{\,i\frac{\partial \hat{U}}{\partial t} = \underbrace{(\hat{H}_{int})_I}_{\equiv\, H_I = \int d^3x\, \frac{\lambda}{4!}\, \phi_I^4}\, \hat{U} = \hat{H}_I \hat{U}\,}$$

$$H_{int} = \frac{\lambda}{4!}\int \phi^4\, d^3x \qquad \phi_I^4$$

$$(H_{int})_I = \frac{\lambda}{4!}\int d^3x\; \overbrace{e^{iH_0(t-t_0)}\, \phi\,\phi\,\phi\,\phi\, e^{-iH_0(t-t_0)}}^{\phi_I^4}$$

$$\underbrace{e^{-iH_0}\, e^{iH_0}}\ \underbrace{\ }\ \underbrace{\ }\ \underbrace{\ } \qquad \phi_I\ \phi_I$$

$\hat{U}(t-t_0) = e^{-i(t-t_0)H_I}$ when H_I has no t

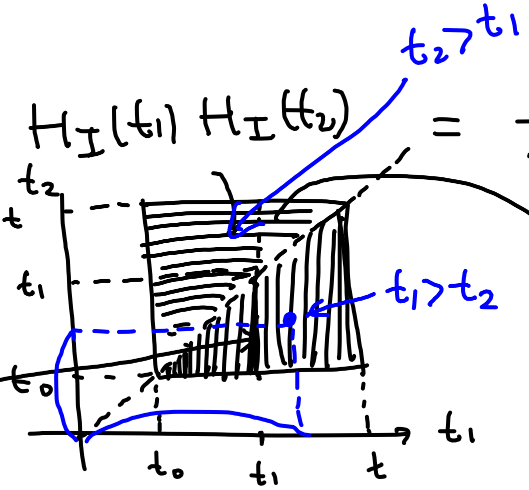
Dyson series

$$\hat{U}(t,t_0) = \mathbb{1} - i\int_{t_0}^{t} H_I(t_1)\,dt_1 + (-i)^2 \underbrace{\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \; H_I(t_1)\,H_I(t_2)}$$

$$+ (-i)^3 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_2} dt_3 \, H_I(t_1)\,H_I(t_2)\,H_I(t_3) + \cdots$$

$$i\frac{\partial \hat{U}}{\partial t} = H_I(t) - i\int_{t_0}^{t} dt_2 \; \underline{H_I(t)}\; H_I(t_2) + \cdots = H_I \underbrace{\left(\mathbb{1} - i\int_{t_0}^{t} dt_1\, H_I(t_1) + \cdots\right)}_{\hat{U}}$$

$$I = \underline{\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2} \; H_I(t_1)\,H_I(t_2) = \frac{1}{2}\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \; T\left(H_I(t_1)\,H_I(t_2)\right)$$



t_2

$$\int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \; H_I(t_2)\,H_I(t_1) \overset{t_1 \leftrightarrow t_2}{=} I$$

$$2I = \underbrace{\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2} \left.\begin{array}{l} \underline{(t_1 > t_2)} \\ H(t_1)\,H(t_2) \\ t_2 > t_1 \; H(t_2)\,H(t_1) \end{array}\right\}$$

$$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots\cdots \int_{t_0}^{t_{n-1}} dt_n \; H_I(t_1)\cdots H_I(t_n) = \frac{1}{n!}\int\!\!\int_{t_0}^{t}\!\!\cdots dt_1\cdots dt_n\, T\big(H_I(t_1)\cdots H_I(t_n)\big)$$

$$U_I(t,t_0) = \mathbb{1} - i\int_{t_0}^{t} H_I(t_1)\,dt_1 + \frac{(-i)^2}{2!}\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \; T\big(H(t_1)H(t_2)\big) + \cdots$$

$$\boxed{U_I(t,t_0) = T\left\{ e^{-i\int_{t_0}^{t} H_I(t_1)\,dt_1} \right\}}$$

$$\underline{H_0|0\rangle = 0|0\rangle} \quad , \; H|n^0\rangle = E_n^0|n^0\rangle$$

$$E_n^0 > 0$$

$$e^{-iTH_0}\underset{\sim}{|0\rangle} = |0\rangle$$

$$H|\Omega\rangle = E_0|\Omega\rangle \longrightarrow H|n\rangle = E_n|n\rangle$$

$$e^{-iTH}|0\rangle = \sum_{n=\Omega} \underbrace{e^{-iTH}|n\rangle}_{e^{-iTE_n}|n\rangle}\langle n|0\rangle = e^{-iTE_0}|\Omega\rangle\langle\Omega|0\rangle + \sum_{n\neq\Omega} e^{-iTE_n}|n\rangle\langle n|0\rangle$$

$\infty(1-i\varepsilon)$ $\varepsilon>0$

$\downarrow$

$$e^{-iTE_0}\left(\langle\Omega|0\rangle|\Omega) + \cancel{\sum_{n\neq 0} e^{-iT(\overbrace{E_n-E_0}^{\overset{0}{\wedge}})}|n\rangle\langle n|0\rangle}\right)$$

$$\parallel$$

$$e^{-i\underset{\substack{\uparrow\\ \infty(1-i\varepsilon)}}{T}(\underbrace{E_n-E_0}_{\overset{\vee}{0}})} \longrightarrow 0 = e^{-\varepsilon\cdot\infty}$$

$$\therefore\ |\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0T}\langle\Omega|0\rangle\right)^{-1}\cdot e^{-iHT}|0\rangle$$

$$= \lim_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0\overbrace{(T+t_0)}^{t_0-(-T)}}\langle\Omega|0\rangle\right)^{-1}\cdot e^{-iH\overbrace{(T+t_0)}^{t_0-(-T)}}\underset{\substack{\uparrow\\ \underbrace{e^{iH_0(t_0-(-T)}}_{U(t_0,-T)}\underbrace{e^{-iH_0(t_0-(-T))}|0\rangle}_{|0\rangle}}}{\mathbb{1}}\,|0\rangle$$

$$\therefore\ |\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)}\left(e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle\right)^{-1} U(t_0,-T)|0\rangle$$

$$\langle\Omega| = \quad '' \quad \underline{\langle 0|U(T,t_0)}\ \underline{\left(e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle\right)^{-1}}$$

$$\Rightarrow\ \langle\Omega|\Omega\rangle = 1 = \underbrace{e^{-iE_0 2T}|\langle 0|\Omega\rangle|^2}\ \langle 0|\underbrace{U(T,t_0)U(t_0,-T)}_{U(T,-T)}|0\rangle$$

$$\langle\Omega|\,\phi(x)\,\phi(y)\,|\Omega\rangle$$

$$\left(e^{-iE_0 2T}\,|\langle 0|\Omega\rangle|^2\right)^{-1} = \langle 0|U(T,-T)|0\rangle$$

$$= \left(e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle\right)^{-1} \qquad \left(e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle\right)^{-1}$$

$$\langle 0|\underbrace{U(T,t_0)U^{\dagger}(x_0,t_0)}_{U(T,x_0)}\,\phi_I(x)\,\underbrace{U(x_0,t_0)\;U^{\dagger}(y_0,t_0)}_{U(x_0,y_0)}\,\phi_I(y)\,\underbrace{U(y_0,t_0)\;U(t_0,-T)}_{U(y_0,-T)}|0\rangle$$

$$U(y_0,-T)$$

$$T\left\{e^{-i\int_{x_0}^{T}H_I\,dt'}\right\}$$

$$1 = \langle\Omega|\Omega\rangle$$

$x_0 > y_0$ → 일반화

$$\therefore\ \langle\Omega|T(\phi(x)\,\phi(y))|\underset{\sim}{\Omega}\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0|T\{U(\overset{''}{T},x_0)\phi_I(x)U(\overset{d}{x_0},y_0)\,\phi_I(y)U(y_0,-T)\}|0\rangle}{\langle 0|\,U(T,-T)\,|0\rangle}$$

$$T\{U(\overset{''}{T},x_0)\phi_I(x)U(\overset{d}{x_0},y_0)\,\phi_I(y)U(y_0,-T)\} = T\Big\{\phi_I(x)\,\phi_I(b)\;\underbrace{U(T,x_0)\,U(x_0,y_0)\,U(y_0,-T)}_{U(T,-T)}\Big\}$$

$$\overset{''}{\;}\quad T\left(e^{-i\int_{-T}^{T}H_I(t')\,dt'}\right)$$

$$\boxed{\langle \Omega | T\{ \overset{\downarrow}{\phi}(x)\, \overset{\downarrow}{\phi}(y) \} | \Omega \rangle = \lim_{T \to \infty(1-i\varepsilon)} \frac{\langle 0 | T\{ \phi_I(x)\, \phi_I(y)\, \underset{\sim\sim\sim}{e^{-i\int_{-T}^{T} H_I(t')\,dt'}} \} | 0 \rangle}{\langle 0 |\, \underset{\sim\sim\sim}{e^{-\int_{-T}^{T} H_I(t')\,dt'}}\, | 0 \rangle}}$$

$$\phi_I(x) = \int \frac{d^3\bar{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_{\vec{p}}\, e^{-ip\cdot x} + a_{\vec{p}}^{\dagger}\, e^{ip\cdot x} \right)$$

$$\langle \Omega | T\{ \phi(x_1) \cdots \phi(x_n) \} | \underset{\sim\sim\sim}{\Omega} \rangle = \lim_{T \to \infty(1-i\varepsilon)} \frac{\underset{\sim}{\langle 0 |}\, \overset{\downarrow}{T}\{ \phi_I(x_1) \cdots \phi_I(x_n)\, e^{-i\int_{-T}^{T} H_I(t')\,dt'} \} \underset{\sim}{|0\rangle}}{\langle 0 |\, \underset{\sim\sim\sim}{e^{-\int_{-T}^{T} H_I(t')\,dt'}}\, | 0 \rangle}$$

4.3. Wick theorem

Time ordered product $\longleftrightarrow$ Normal ordered product

$T(\cdots\cdot)$ $\qquad$ $N(\cdots)$

Normal ordered product

$$\phi_I(x) = \int \frac{d^3\vec{p}}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\left(\underset{\sim}{a_{\vec{p}}}\, e^{-ip\cdot x} + \underset{\sim}{a^\dagger_{\vec{p}}}\, e^{ip\cdot x}\right) = \phi^+ + \phi^-$$

ϕ^-

ϕ^+

$$\phi^+|0\rangle = 0 \quad (\because a_{\vec{p}}|0\rangle = 0)$$

$$\langle 0|\phi^- = 0 \quad (\because \langle 0|a^\dagger_{\vec{p}} = 0)$$

$$N\Big(\underline{\overbrace{(\phi^+(x) + \phi^-(x))}^{\phi(x)}}\ \underline{\overbrace{(\phi^+(y) + \phi^-(y))}^{\phi(y)}}\Big) = \underline{N}\Big(\underline{\phi^+(x)\phi^+(y)} + \underline{\phi^-(x)\phi^+(y)} + \phi^+(x)\phi^-(y) + \underline{\phi^-(x)\phi^-(y)}\Big)$$

$$= \underline{\phi^+(x)\phi^+(y)} + \underline{\phi^-(x)\phi^+(y)} + \underset{\sim}{\phi^-(y)\phi^+(x)} + \underline{\phi^-(x)\phi^-(y)}$$

(ex) $N(a_{\vec{k}_1} a^\dagger_{\vec{k}_2} a^\dagger_{\vec{k}_3}) = a^\dagger_{\vec{k}_2} a^\dagger_{\vec{k}_3} a_{\vec{k}_1}$

$$T\Big(\underline{\overbrace{(\phi^+(x)+\phi^-(x))}^{\phi(x)}}\,\underline{\overbrace{(\phi^+(y)+\phi^-(y))}^{\phi(y)}}\Big) = \underline{\phi^+(x)\,\phi^+(y)} + \underline{\phi^-(x)\,\phi^+(y)} + \underline{\phi^+(x)\,\phi^-(y)} + \phi^-(x)\phi^-(y)$$

$\underline{x^0 > y^0}$

$$= N\big(\phi(x)\,\phi(y)\big) + \underbrace{\phi^+(x)\,\phi^-(y) - \phi^-(y)\,\phi^+(x)}$$

$$[\phi^+(x), \phi^-(y)] = [\underline{\phi(x), \phi(y)}]$$

$$\overset{\shortparallel}{\overbrace{\phi(x)\,\phi(y)}} \quad \text{"contraction"}$$

$\underline{y^0 > x^0}$

$$T(\phi(x)\,\phi(y)) = \phi(y)\,\phi(x) = N(\phi(x)\,\phi(y)) + [\phi^+(y), \phi^-(x)]$$

$$\swarrow$$

$$[\phi(y), \phi(x)]$$

$$\langle 0|\,[\underline{\phi(x),\phi(y)}]\,|0\rangle = [\phi(x), \phi(y)]$$

$$\langle 0|\,\underline{\phi^+(x)}\ \underline{\phi^+(y)}\,|0\rangle$$

$$\phi^+ \qquad \phi^-$$

$$T\big([\phi(x), \phi(y)]\big)$$

$$= D_F(x-y)$$

$$\boxed{T\big(\phi_I(x_1)\cdots\phi_I(x_n)\big) = N\big(\phi_I(x_1)\cdots\phi_I(x_n) + \text{all possible contractions}\big)}$$

(ex) $T(\phi_I(x)\,\phi_I(y)) = N\Big(\phi_I(x)\,\phi_I(y) + \overbrace{\phi_I(x)\,\phi_I(y)}^{=D_F(x-y)}\Big)$

$\phi_I(x_n) \equiv \phi_n$

$$T\big(\phi_1\phi_2\phi_3\phi_4\big) = N\Big(\underbrace{\phi_1\phi_2\phi_3\phi_4}_{\sim\sim\sim} + \overbrace{\phi_1\underset{\sim}{\phi_2}\phi_3}\underset{\sim}{\phi_4} + \overbrace{\phi_1\phi_2}^{D_F(x_1-x_2)}\phi_3\phi_4 + \overbrace{\phi_1\phi_2\phi_3\phi_4}^{D_F(x_1-x_4)}$$

$$\overbrace{\phi_1\phi_2\phi_3}\phi_4 = \underbrace{\phi_2\phi_4}_{\text{op.}}\ \underbrace{\overbrace{\phi_1\phi_2}}_{\#}$$

$$+\ \phi_1\overbrace{\phi_2\phi_3}\phi_4 + \phi_1\overbrace{\phi_2\phi_3\phi_4} + \phi_1\phi_2\overbrace{\phi_3\phi_4} + \overbrace{\phi_1\phi_2}\,\overbrace{\phi_3\phi_4} + \overbrace{\phi_1\overbrace{\phi_2\phi_3}\phi_4}^{D_F(x_1-x_4)D_F(x_2-x_3)} + \overbrace{\phi_1\phi_2\phi_3}\overbrace{\phi_2\phi_3\phi_4}\Big)$$

= fully contract

Correlation function.

$$\langle 0|N\{\qquad\qquad\}|0\rangle = \text{fully contracted terms only}$$

$$\langle 0|T(\phi_1\phi_2\phi_3\phi_4)|0\rangle = D_F(x_1-x_2)D_F(x_3-x_4) + D_F(x_1-x_4)D_F(x_2-x_3)$$
$$+ D_F(x_1-x_3)D_F(x_2-x_4)$$

4.4. Feynman Diagrams

- real space form
- momentum space form

$D_F(x-y) =$ 

$= \wick{\c{\phi}(x)\c{\phi}(y)}$

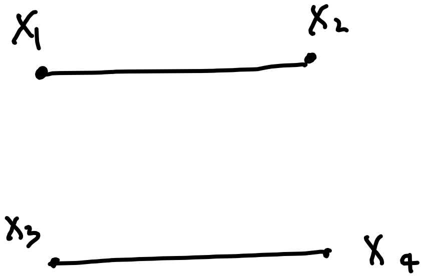
$$= \int \frac{d^4p}{(2\pi)^4}\frac{i}{p^2-m^2+i\varepsilon}\, e^{-ip\cdot(x-y)}$$

$$\langle 0|T(\phi_1\phi_2\phi_3\phi_4)|0\rangle$$

$$= D_F(x_1-x_2)D_F(x_3-x_4) + D_F(x_1-x_4)D_F(x_2-x_3)$$

$$+ D_F(x_1-x_3)D_F(x_2-x_4)$$

$=$



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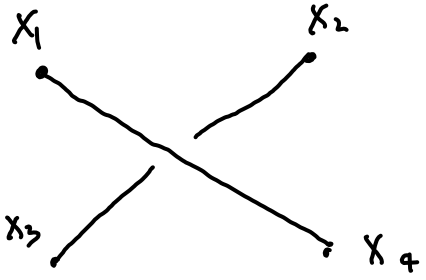

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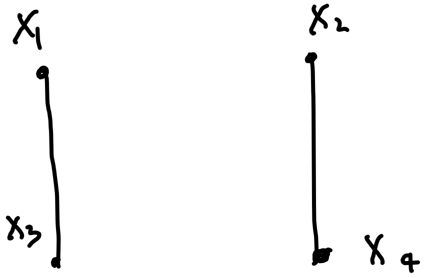


$$\boxed{\langle\Omega|T\{\phi(x)\,\phi(y)\}|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0|T\{\phi_I(x)\,\phi_I(y)\,\underline{e^{-i\int_{-T}^{T}H_I(t')\,dt'}}\}|0\rangle}{\langle 0|\,\underline{e^{-\int_{-T}^{T}H_I(t')\,dt'}}\,|0\rangle}}$$

$$\int d^3x'\,\frac{\lambda}{4!}\,\phi_I^4 \rightsquigarrow \int d^4x'\,\frac{\lambda}{4!}\,\phi_I(x')^4$$

$$\langle 0|T\{\phi_I(x)\,\phi_I(y)\,\underline{e^{-i\int_{-T}^{T}\overbrace{H_I(t')\,dt'}}}\}|0\rangle$$

← $\lambda \ll 1$ → Taylor expasion

$\phi \equiv \phi_I$

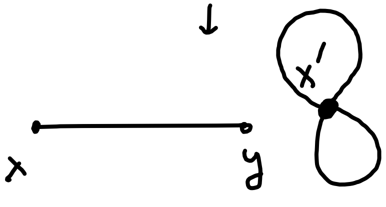
$$= \langle 0|T(\phi(x)\,\phi(y))|0\rangle + \frac{(-i)\lambda}{4!}\langle 0|T\left(\underline{\phi(x)\,\phi(y)}\int d^4x'\,\phi(x')^4\right)|0\rangle$$

$$+\frac{1}{2!}\left(\frac{-i\lambda}{4!}\right)^2\langle 0|T\left(\phi(x)\,\phi(y)\int d^4x'\,\phi(x')^4\int d^4x''\,\phi(x'')^4\right)|0\rangle$$

$$+\cdots$$

$=$

$$+\frac{(-i)\lambda}{4!}\int d^4x'\left(\phi(x)\,\phi(y)\,\phi'\,\phi'\,\phi'\,\phi' + \phi(x)\,\phi(y)\,\underbrace{\phi'\,\phi'\,\phi'\,\phi'}_{x'}\right.$$



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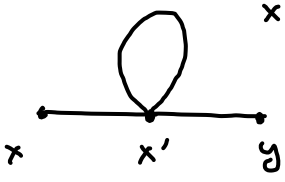


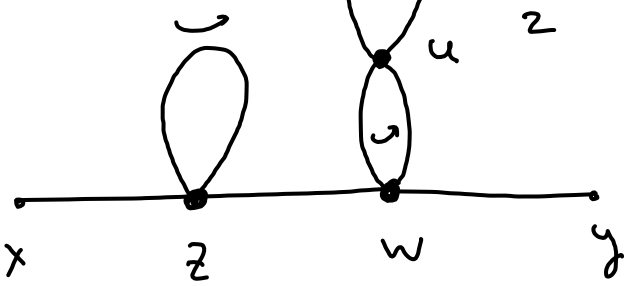
a simple way to compute Combinatorics

$$\lambda^3 : \quad \langle 0 | N \left(\phi(x)\phi(y) \int d^4z\, \phi\phi\phi\phi \left(\int d^4w\, \phi\phi\phi\phi \int d^4u\, \phi\phi\phi\phi \right) \right) | 0 \rangle$$

$$\frac{1}{3!}\left(\frac{-i\lambda}{4!}\right)^3 \times$$

$$3! \qquad \underset{\sim}{4\cdot 3} \quad \frac{4!}{2} \qquad \frac{\underset{\sim}{4\cdot3\cdot2}}{4!} \qquad \frac{\underset{\sim}{4\cdot 3}}{\frac{4!}{2}} \cdot \frac{1}{2}$$

$$\frac{(-i\lambda)^3}{\underbrace{\boxed{2\cdot 2\cdot 2}}_{8}} \cdot$$

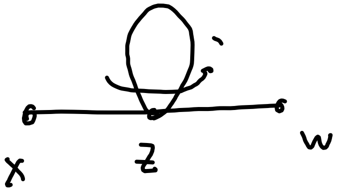


$S = 8$ $\Rightarrow$ $\frac{\lambda}{4!}\phi^4 \to \lambda\phi^4$

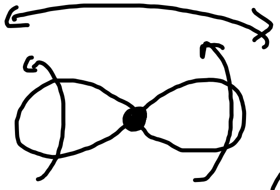
$$\downarrow$$

$$\frac{1}{\cancel{3!}}\,\cancel{3!}\cdot\frac{\lambda^3}{S}$$

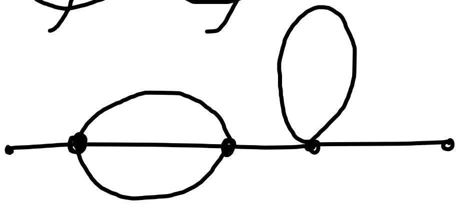
S factor.



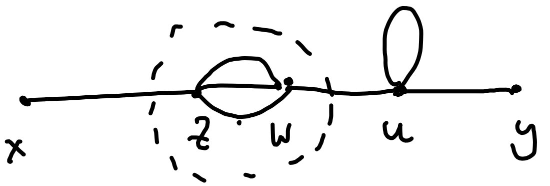
$S = 2$



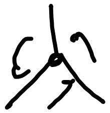
$S = 2\cdot 2\cdot 2 = 8$



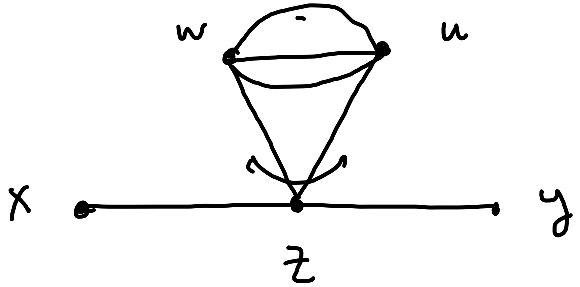
$$\langle 0 | N\left(\phi(x)\phi(y) \int d^4z\, \phi\phi\phi\phi \int d^4w\, \phi\phi\phi\phi \int d^4u\, \phi\phi\phi\phi \right) | 0 \rangle$$



$$= \frac{(-i\lambda)^3}{3!\cdot 2}$$

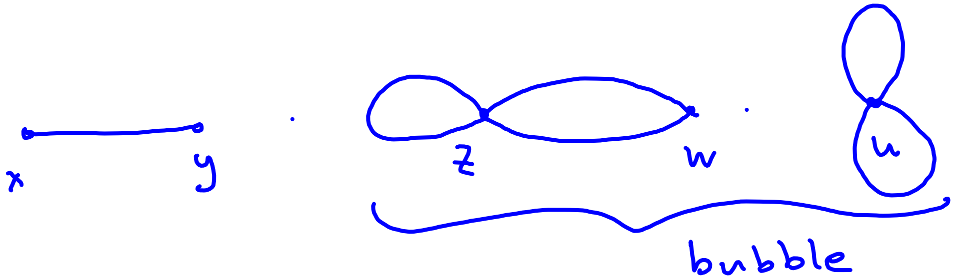


$S=3!$



$$= \frac{(-i\lambda)^3}{3!\cdot 2}$$

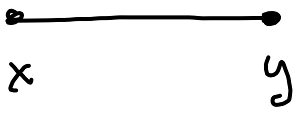
$$\langle 0 | N\left(\phi(x)\phi(y) \int d^4z\, \phi\phi\phi\phi \int d^4w\, \phi\phi\phi\phi \int d^4u\, \phi\phi\phi\phi \right) | 0 \rangle$$



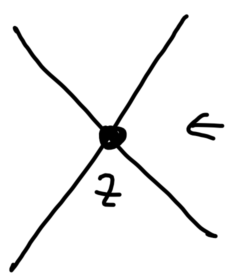
real space Feynman Rule for scalar field (ϕ^4)

$$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i\, e^{-ip(x-y)}}{p^2 - m^2 + i\epsilon}$$

1. propagator

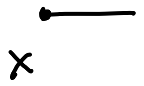


2. vertex



$\leftarrow -i\lambda \int d^4z$

3. external line: $\langle \Omega | T(\phi(\underset{\cdot}{x})\phi(\underset{\cdot}{y})) | \Omega \rangle$

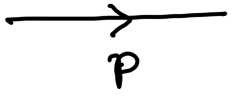


: 1

4. Divide by symmetry factor S

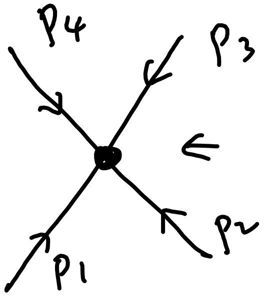
momentum space Feynman rule for scalar field (ϕ^4)

1. propagator



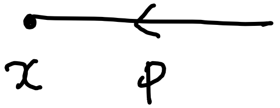
$$\tilde{D}_F(p) = \frac{i}{p^2-m^2+i\varepsilon}$$

2. vertex



$\leftarrow -i\lambda \leftarrow \left(\underline{\delta^{(4)}(\Sigma p_i)} \right)$

3. external line:



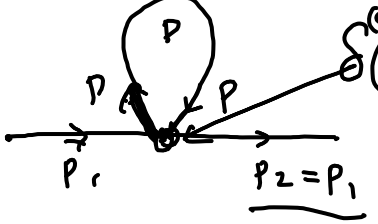
$= e^{-ip\cdot x}$

H.W.
Prob.
4.1.

4. Divide by symmetry factor S

(loop momenta)

5. integral over "$\underline{\text{undetermined}}$" momenta $\int \frac{d^4p}{(2\pi)^4}$

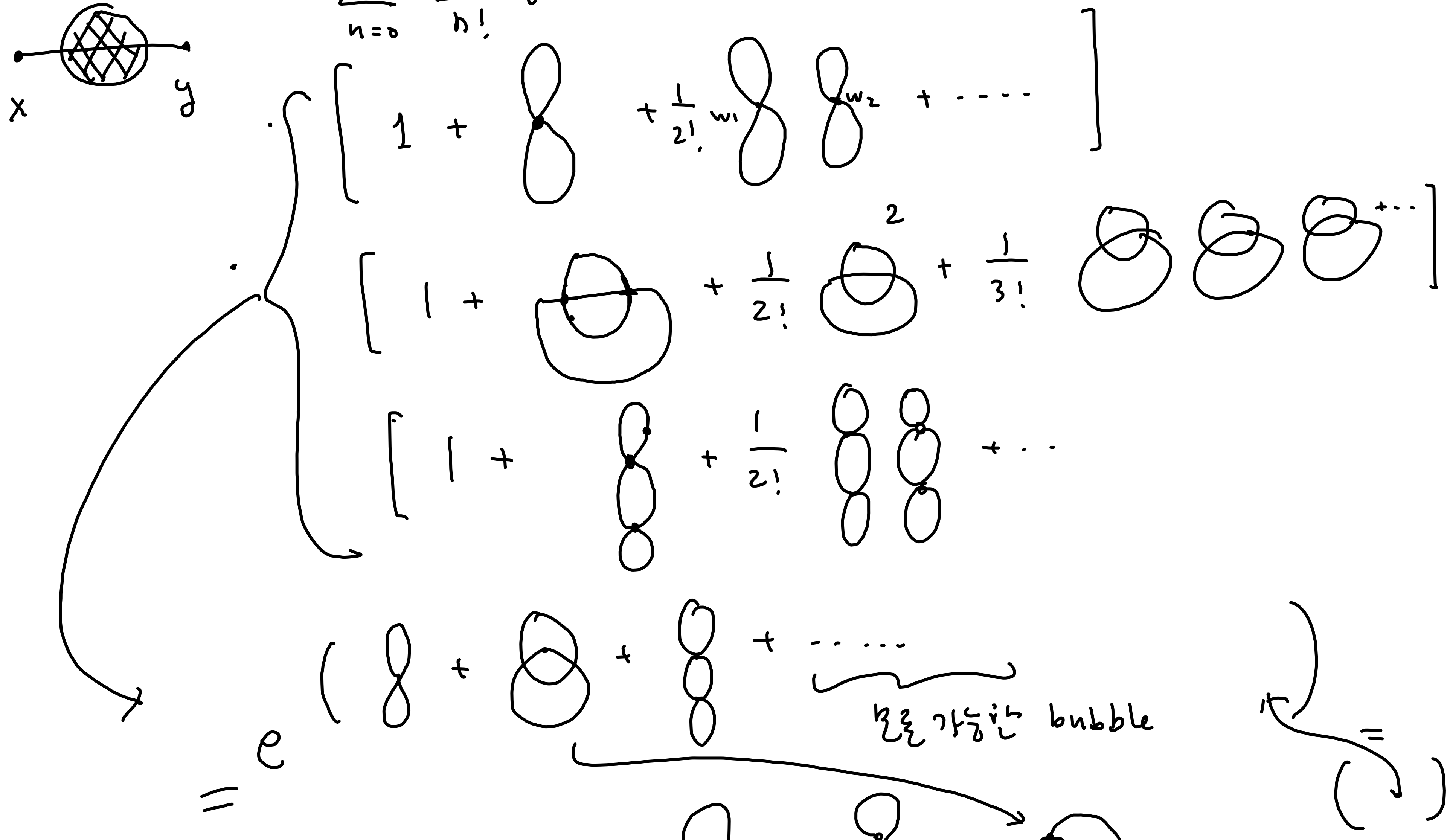


$\delta^{(4)}(p_1-p_2+\cancel{(-p)}+\cancel{p}) =$

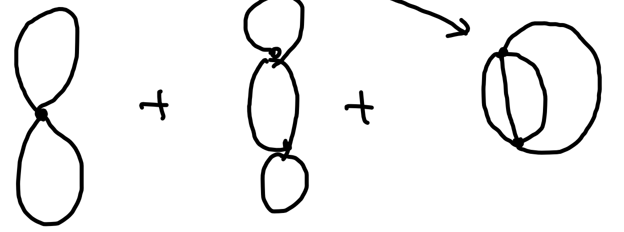
$$\langle 0|T\{\underline{\phi(x)\phi(y)}\ e^{-i\lambda\int\phi^4 d^4_w}\}|0\rangle$$

$$\Downarrow$$

$$\sum_{n=0}^{\infty}\frac{(-i\lambda)^n}{n!}\int d^4w_1\cdots\int d^4w_n\ \phi^4(w_1)\ \phi^4(w_2)\ \cdots\ \phi^4(w_n)$$



$$\langle 0|T\{e^{-i\lambda\ \phi^4 d^4_w}\}|0\rangle = 1 +$$

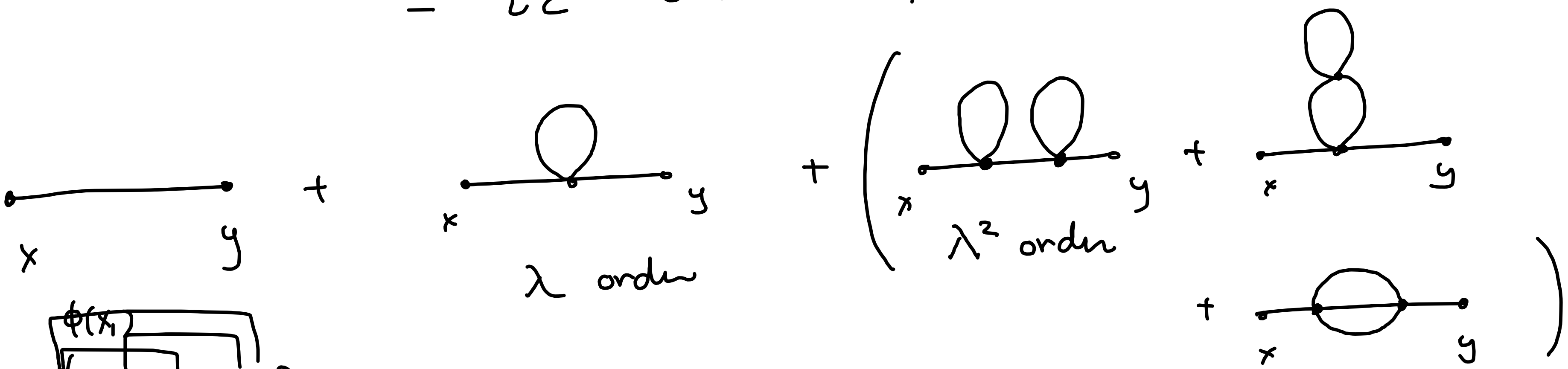


$$+\cdots = e$$

$$\langle\Omega|T\{\phi(x)\phi(y)\}|\Omega\rangle = \lim_{T\to\infty(1-i\epsilon)} \underbrace{\frac{\langle 0|T\{\phi_I(x)\phi_I(y)\, e^{-i\int_{-T} H_I(t')dt'}\}|0\rangle}{\langle 0|\, e^{-\int_{-T}^{T} H_I(t')dt'}\,|0\rangle}}_{\text{모든 bubble은 약분}}$$

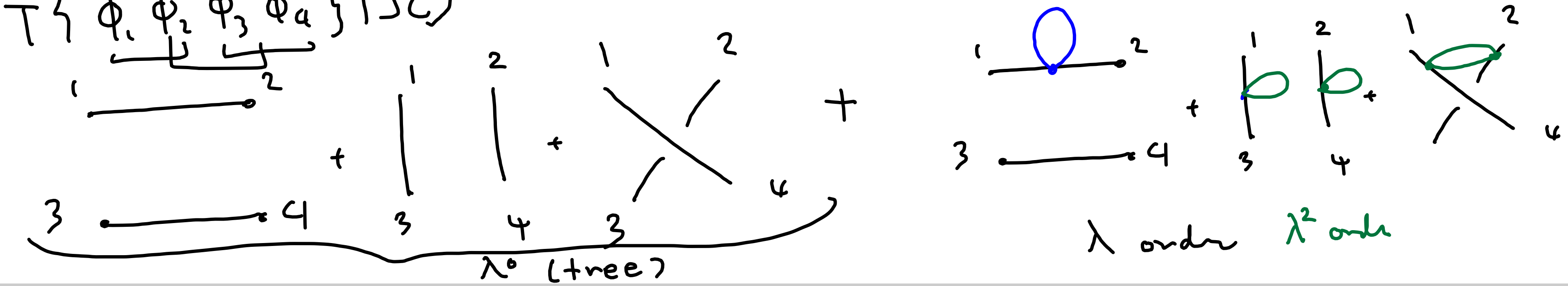
= 모든 연결된 Feynman Diagram.

=

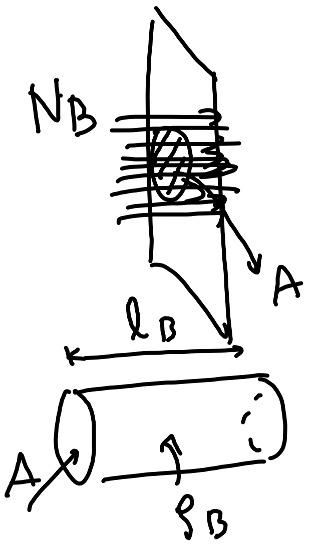


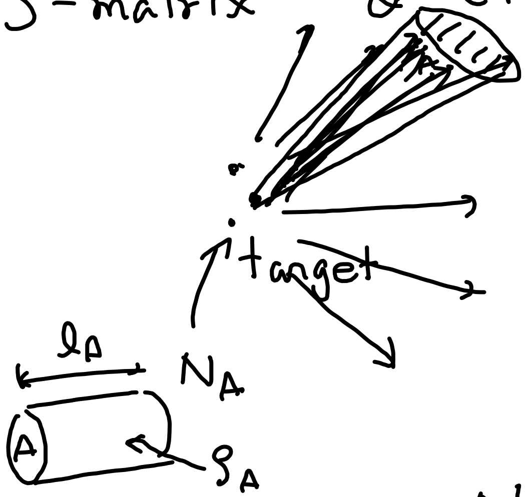
$\phi(x_1)$

$$\langle\Omega|T\{\phi_1\phi_2\phi_3\phi_4\}|\Omega\rangle$$



4.5. S-matrix & cross section





$$\underbrace{\overset{N}{\text{scattering event 수}}} \propto \text{flux} \cdot \text{target의 갯수}$$

↓

$$\underset{\uparrow}{N} = \underset{\underset{\text{area}}{\uparrow}}{\sigma} \, \underline{\frac{N_B}{A} N_A}$$

$N_B = \rho_B A l_B$

$N_A = \rho_A A l_A$

$$\underset{\underset{\text{total}}{\uparrow}}{\sigma} = \frac{N}{\frac{N_A N_B}{A}} = \frac{N}{\rho_A \rho_B l_A l_B A}$$

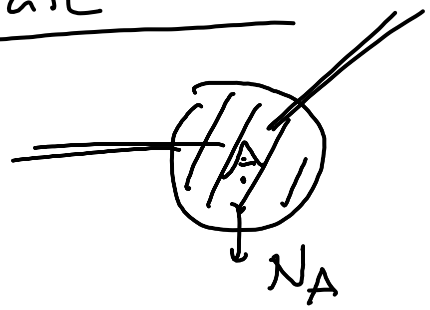
$$d\sigma = \frac{\boxed{dN_A}}{\frac{N_A N_B}{A}} \leftarrow \text{미분 산란 단면적}$$

$\Downarrow$

theory ?

Decay rate Γ

when 입자가 unstable 할 때.



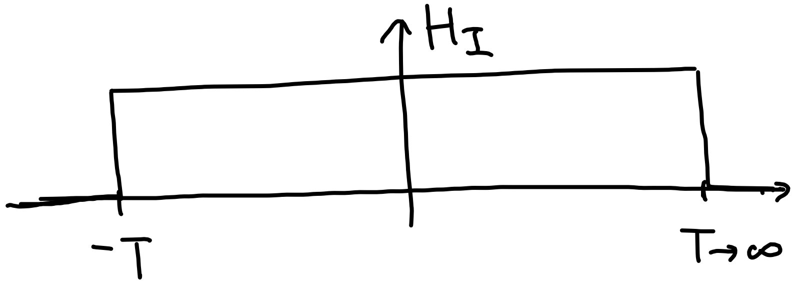
$$\Gamma = \frac{\#\text{ of decays per unit time}}{N_A} = [\text{time}]^{-1}$$

$$\tau = \frac{1}{\Gamma}$$

$$\boxed{\underbrace{S\ (\text{cross section})},\quad \Gamma \iff H_I = ?}$$

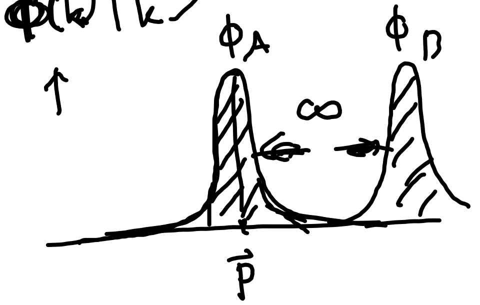
scattering state in QFT.

one-particle state with mom. $\vec{k}$: $|\vec{k}\rangle_0 = \frac{1}{\sqrt{2E_{\vec{k}}}} a^\dagger_{\vec{k}} |0\rangle$



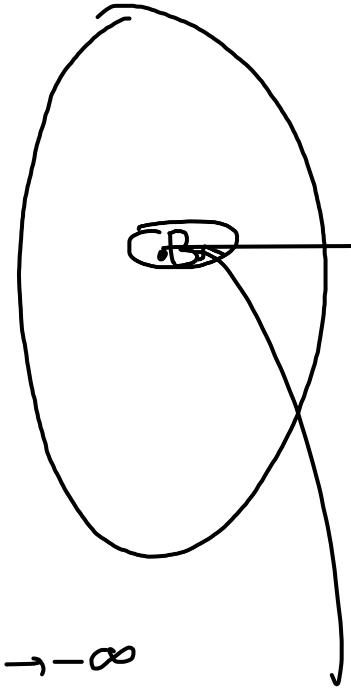
$$|\phi\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \underset{\uparrow}{\phi(\vec{k})} |\vec{k}\rangle$$

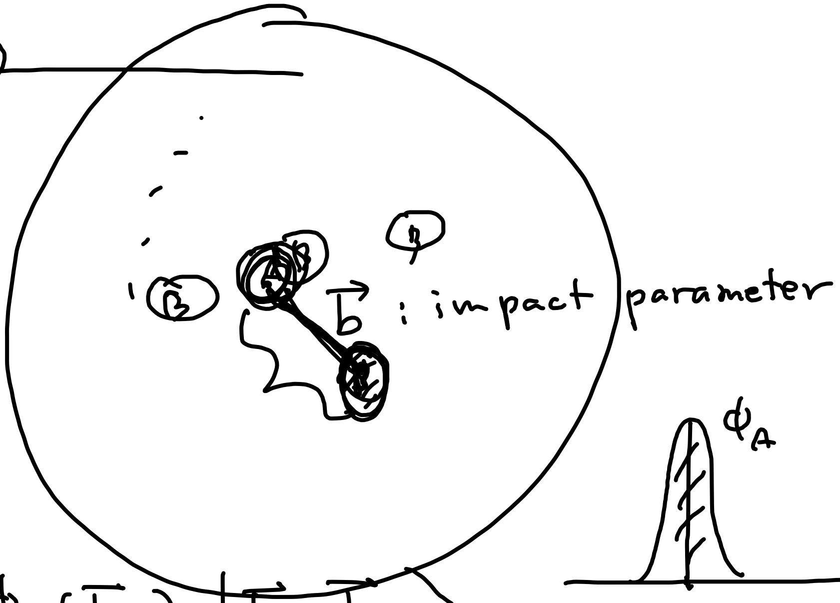
$$\langle \underbrace{\phi_1 \phi_2 \cdots\cdots}_{\text{future}} | \underbrace{\phi_A \phi_B}_{\text{past}} \rangle$$



asymptotic states

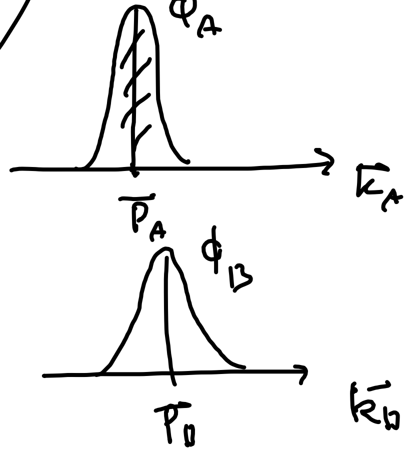
$\exists$



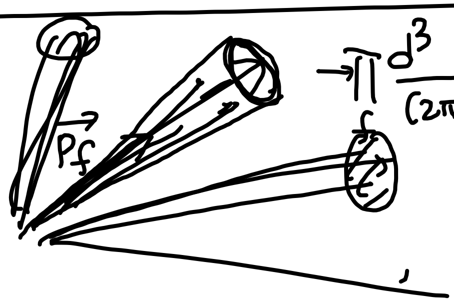


$T \to -\infty$

$$|\phi_A \phi_B\rangle_{in} = \int \frac{d^3k_A}{(2\pi)^3} \int \frac{d^3 k_B}{(2\pi)^3} \frac{e^{-i\vec{b}\cdot\vec{k}_B}}{\sqrt{2E_A 2E_B}} \phi_A(\vec{k}_A)\, \phi_B(\vec{k}_B)\, |\vec{k}_A\, \vec{k}_B\rangle_{in}$$



$T \to \infty$



$$\rightarrow \prod_f \frac{d^3 p_f}{(2\pi)^3} \frac{1}{\sqrt{2E_f}} \phi_f(\vec{p}_f)$$

$${}_{out}\langle \phi_1\, \phi_2 \cdots | = \prod_f \frac{d^3 p_f}{(2\pi)^3} \frac{1}{\sqrt{2E_f}} \phi_f(\vec{p}_f) \;\; {}_{out}\langle \vec{p}_1, \vec{p}_2 \cdots \vec{p}_f \cdots |$$

$${}_{out}\langle \phi_1\, \phi_2 \cdots | \phi_A\, \phi_B \rangle_{in} = \int \cdots \int \cdots \prod_f \cdots \quad \underbrace{\underset{T\to\infty}{\underbrace{{}_{out}\langle \vec{p}_1\, \vec{p}_2 \cdots |}} \;\underset{T\to-\infty}{\underbrace{\vec{k}_A\, \vec{k}_B \rangle_{in}}}}_{S\text{-matrix}}$$

$$= \underbrace{\langle \vec{p}_1\, p_2 \cdots |\, S \,| \vec{k}_A\, \vec{k}_B \rangle}_{S\text{-matrix element}}$$

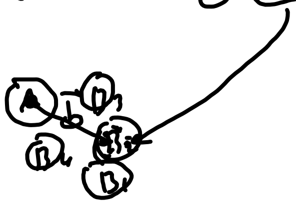
$$S = \mathbb{1} + i \underbrace{\mathbb{T}}_{\substack{T\text{-matrix} \\ (\text{interaction 효과})}}$$

$$\boxed{\langle \vec{p}_1\, \vec{p}_2 \cdots |\, i\mathbb{T} \,| \vec{k}_A\, \vec{k}_B \rangle = i\mathcal{M}(k_A, k_B, \{p_i\})(2\pi)^4 \delta^{(4)}\Big(k_A + k_B - \sum_i p_i\Big)}$$

$$k_A^0 = \sqrt{m_A^2 + \vec{k}_A^2}\,, \quad p_2^0 = \sqrt{m_2^2 + \vec{p}_2^{\,2}} \quad \cdots$$

$$\underset{\substack{\uparrow \\ \text{prob.}}}{P}(AB \to 1\, 2 \cdots n) = \prod_f \left(\frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} \right) \Big| \underbrace{{}_{out}\langle \phi_1\, \phi_2 \cdots | \phi_A\, \phi_B \rangle_{in}}\Big|^2$$

$$N = \sum_i P_{B_i} = \int \underbrace{d^2 b}\; \underset{\substack{\uparrow \\ \text{flux}}}{n_B}\, P(\vec{b})$$



$$\sigma = \frac{N}{\underbrace{\frac{N_B \cdot 1}{A}}_{n_B =}} = \int d^2 b\, P(\vec{b})$$

$$\therefore\ d\sigma = \prod_f \left(\frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right) \underset{\sim\sim}{\int d^2b}\ \left| \int \frac{d^3k_A d^3k_B}{(2\pi)^6\sqrt{E_A E_B}}\ \underbrace{{}_{out}\langle \vec{p}_1\ \vec{p}_2 \cdots | \vec{k}_A \vec{k}_B\rangle_{in}}_{{}_{out}\langle\{\vec{p}_f\}|\{\vec{k}_i\}\rangle_{in}}\ e^{i\vec{b}\cdot\vec{k}_B}\right|^2$$

$$\int d^2b \left| \int \frac{d^3k_A d^3k_B}{(2\pi)^6\sqrt{E_A E_B}}\ \underbrace{{}_{out}\langle \vec{p}_1\ \vec{p}_2 \cdots | \vec{k}_A \vec{k}_B\rangle_{in}}_{{}_{out}\langle\{\vec{p}_f\}|\{\vec{k}_i\}\rangle_{in}}\ e^{i\vec{b}\cdot\vec{k}_B}\right|^2 = \underbrace{\int \frac{d^3k_A\, d^3k_B}{(2\pi)^6\sqrt{2E_{k_A}2E_{k_B}}}}\ \underbrace{\int \frac{d^3\bar{k}_A\, d^3\bar{k}_B}{(2\pi)^6\sqrt{2E_{\bar{k}_A}2E_{\bar{k}_B}}}}\ \underbrace{{}_o\langle\{\vec{p}_f\}|\{\vec{k}_i\}\rangle_i}_{\propto\,\delta^{(4)}(k_A+k_B-\sum_f p_f)}\ \underbrace{{}_i\langle\{\bar{k}_i\}|\{\vec{p}_f\}\rangle_o}_{\propto\,\underbrace{\delta^{(4)}(\bar{k}_A+\bar{k}_B-\sum_f p_f)}_{4개}} \int d^2b\, e^{-i\vec{b}\cdot\bar{k}_B}\, e^{i\vec{b}\cdot\vec{k}_B}$$

$$\langle \vec{p}_1\ \vec{p}_2 \cdots | iT | \vec{k}_A \vec{k}_B\rangle = \underbrace{i\mathcal{M}(k_A, k_B \to \{p_f\})}\ \underbrace{(2\pi)^4\delta^{(4)}(k_A+k_B-\sum_f p_f)}$$

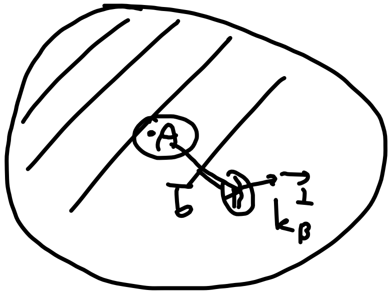
$$\int d^2b\ e^{-i\vec{b}\cdot(\underbrace{\vec{k}_B-\vec{\bar{k}}_B}_{\vec{k}_B^{\perp}-\vec{\bar{k}}_B^{\perp}})} = \underbrace{(2\pi)^2\,\delta^{(2)}(\vec{k}_B^{\perp}-\vec{\bar{k}}_B^{\perp})}_{2개}$$

6개의

delta-fuct

⇓

$$\int d^3\bar{k}_A \int d^3\bar{k}_B$$



$$\int d\bar{k}_A^z \underbrace{\int d\bar{k}_B^z \, \delta\Big(\bar{k}_A^z + \bar{k}_B^z - \sum_f p_f^z\Big)}_{1} \delta\Big(E_{\bar{k}_A} + E_{\bar{k}_B} - \sum_f E_f\Big)$$

$$\underbrace{\bar{k}_B^z = -\bar{k}_A^z} + \sum_f p_f^z$$

$$\boxed{\int dx\, \delta(f(x)) = \frac{1}{|f'(x_0)|} \qquad f(x_0) = 0}$$

$$= \int d\bar{k}_A^z \, \delta\Big(E_{\bar{k}_A} + E_{\bar{k}_B} - \sum_f E_f\Big)$$

$$\frac{d}{d\bar{k}_A^z}\sqrt{\underbrace{m_A^2 + \bar{k}_A^{x\,2} + \bar{k}_A^{y\,2}} + \bar{k}_A^{z\,2}} = \frac{\bar{k}_A^z}{\sqrt{\cdots}} = \frac{\bar{k}_A^z}{E_{\bar{k}_A}}$$

$$= \frac{1}{\left|\underbrace{\frac{\bar{k}_A^z}{E_{\bar{k}_A}}}_{\bar{v}_A^z} - \underbrace{\frac{\bar{k}_B^z}{E_{\bar{k}_B}}}_{\bar{v}_B^z}\right|}$$

$$\frac{dE_{\bar{k}_B}}{d\bar{k}_A^z} = \underbrace{\frac{dE_{\bar{k}_B}}{d\bar{k}_B^z}}_{\frac{\bar{k}_B^z}{E_{\bar{k}_B}}} \underbrace{\frac{d\bar{k}_B^z}{d\bar{k}_A^z}}_{(-1)}$$

$$E = \gamma m$$

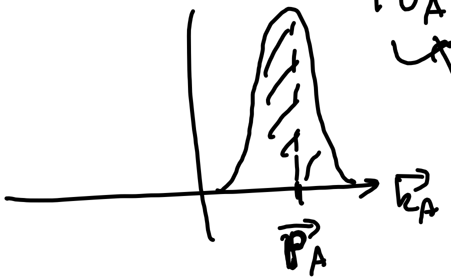
$$\vec{k} = \gamma m \vec{v}$$

$$\frac{\vec{k}}{E} = \vec{v}$$

$$d\sigma = \prod_f\left(\frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right) \underbrace{\int\frac{d^3k_A\,d^3k_B}{(2\pi)^6}\left\{|\phi_A(k_A)|^2|\phi_B(k_B)|^2\right\}}_{\to 1}$$

$$\frac{1}{2E_{p_A}2E_{p_B}}\underbrace{\frac{1}{|v_A-v_B|}|\mathcal{M}(p_A+p_B\to\sum p_f)|^2}(2\pi)^4\delta^{(4)}\Big(p_A+p_B-\sum_f p_f\Big)$$

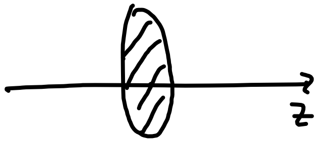
$$\frac{k_A^z}{E_A}\to\frac{p_A^z}{E_A}$$



$$\therefore\quad \boxed{d\sigma = \underbrace{\frac{1}{2E_A2E_B|v_A-v_B|}}\underbrace{\prod_f\left(\frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)}|\mathcal{M}(p_A+p_B\to\sum p_f)|^2\,\underline{(2\pi)^4\delta^{(4)}\Big(p_A+p_B-\sum_f p_f\Big)}}$$

cross section

$$\underline{\langle\vec{p}_1\,\vec{p}_2\cdots|iT|\vec{p}_A\,\vec{p}_B\rangle} = i\underbrace{\mathcal{M}(p_Ap_B\to\{p_f\})}(2\pi)^4\,\underline{\delta^{(4)}\Big(k_A+k_B-\sum_f p_f\Big)}$$



- not Lorentz invariant: $\frac{1}{4}\cdot\frac{1}{|\underset{\substack{\downarrow\\ \underbrace{m_A\gamma_Av_A}_{p_A^z}}}{E_AE_Bv_A} - E_A\underset{\substack{\downarrow\\ p_B^z}}{\underbrace{E_Bv_B}}|} = \frac{1}{|\underbrace{E_Bp_A^z - E_Ap_B^z}|}$ $\quad -\epsilon_{\mu xy\nu}\underset{\text{4-vect}}{p_A^\mu} p_B^\nu$
- boost invariant in 0-z plane

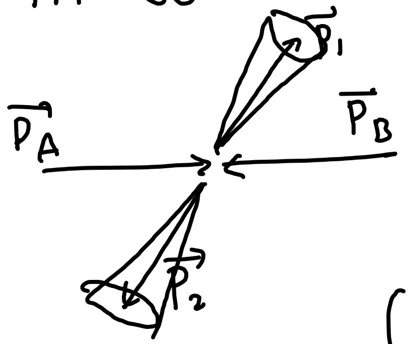
- $A + B \longrightarrow \sum_f f$

- $\underline{2 \longrightarrow 2}$

$$\prod_{f=1,2}\left(\frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right) = \frac{d^3\vec{p}_1}{(2\pi)^6\, 2E_1 2E_2}(2\pi)^4\delta(E_A+E_B-E_1-E_2)$$

$$\underbrace{\vec{p}_1 = -\vec{p}_2} \Rightarrow E_1 \neq E_2$$

in center of mass frame



$$(\vec{p}_A + \vec{p}_B) = 0 = \vec{p}_1 + \vec{p}_2$$

$$d^3\vec{p}_1 = |\vec{p}_1|^2 d|\vec{p}_1|\, d\Omega$$

$$\int \frac{d^3\vec{p}_1}{(2\pi)^6\, 2E_1 2E_2}(2\pi)^4\delta(\underbrace{E_A+E_B}_{\overset{\shortparallel}{E_{cm}}}-E_1-E_2) = \int d\Omega\, \underbrace{\int \frac{|\vec{p}_1|^2\, d|\vec{p}_1|}{4\cdot(2\pi)^2\, E_1E_2}\delta(E_{cm}-E_1-E_2)}$$

$$\left((\mathrm{cf})\ d^3r = r^2 dr\, \underbrace{d\cos\theta\, d\phi}_{d\Omega}\right)$$

$$E_1 = \sqrt{m_1^2 + |\vec{p}_1|^2}$$

$$E_2 = \sqrt{m_2^2 + |\vec{p}_1|^2}$$

$$= \int d\Omega \frac{|\vec{p}_1|^2}{16\pi^2 E_1E_2}\frac{1}{\left|\underbrace{\frac{dE_1}{d|\vec{p}_1|}} + \underbrace{\frac{dE_2}{d|\vec{p}_1|}}\right|}$$

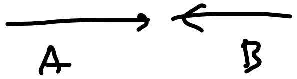
$$\frac{|\vec{p}_1|}{E_1} + \frac{|\vec{p}_1|}{E_2} = \frac{|\vec{p}_1|(\overbrace{E_1+E_2}^{E_{cm}})}{E_1E_2}$$

$$\frac{d\sigma}{d\Omega} = \frac{|\vec{p}_1|}{16\pi^2 E_{cm}} \cdots \Leftarrow \quad = \underbrace{\int d\Omega \frac{|\vec{p}_1|}{16\pi^2 E_{cm}}}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{|\vec{p}_1|}{16\pi^2 E_{cm}} \frac{1}{2E_A 2E_B |v_A - v_B|} |\mathcal{M}(p_A + p_B \rightarrow p_1 + p_2)|^2}$$

if $m_A = m_B = m_1 = m_2 \equiv m$

$$E_A = E_B = E_1 = E_2 = \frac{E_{cm}}{2}$$



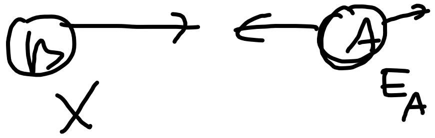
$$|\vec{p}_1| = \underbrace{\gamma m}_{E_1} v_A = E_1 v_A$$

$$v_B = -v_A$$

$$\frac{\cancel{|\vec{p}_1|}}{16\pi^2 E_{cm}} \frac{1}{\cancel{4} \cdot \frac{E_{cm}^2}{\cancel{4}}} \frac{1}{2 \frac{\cancel{|\vec{p}_1|}}{E_1 = \frac{E_{cm}}{2}}} = \frac{1}{64\pi^2 E_{cm}^2}$$

$$\therefore \boxed{\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}(p_A + p_B \rightarrow p_1 + p_2)|^2}{64\pi^2 E_{cm}^2}}$$

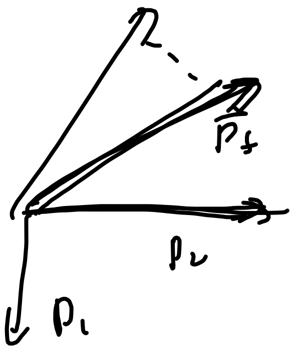
Decay rate



→ 정지 $\vec{p}_A = 0$

$E_A = m_A$

$$d\Gamma = \frac{1}{2m_A}\underline{\prod_f\left(\frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)}\left|M(p_A+p_B \to \sum p_f)\right|^2 \underline{(2\pi)^4\delta^{(4)}(\overset{(m_A,\,\vec{0})}{p_A} - \sum_f p_f)}$$



- when identical particles

$$\downarrow \frac{1}{\prod_i n_i!}$$

4. 6. $\mathcal{M}$

$$\langle \vec{p}_1\, \vec{p}_2 \cdots | i\mathbb{T} | \vec{k}_A\, \vec{k}_B \rangle = \underbrace{i\mathcal{M}(k_A, k_B \to \{p_f\})}\,\underbrace{(2\pi)^4\, \delta^{(4)}\Big(k_A + k_B - \sum_f p_f\Big)}$$

$$\uparrow$$

$$\langle \vec{p}_1\, \vec{p}_2 \cdots | S | \vec{k}_A\, \vec{k}_B \rangle = \underbrace{\langle \vec{p}_1\, \vec{p}_2 \cdots}_{T \to \infty} \; | \vec{k}_A\, \vec{k}_B \rangle_{T \to -\infty}$$

$$= \lim_{T \to \infty} {}_0\langle \vec{p}_1\, \vec{p}_2 \cdots | e^{-iH(2T)} \underbrace{| \vec{k}_A\, \vec{k}_B \rangle_0}$$

$$\propto \lim_{T \to \infty} {}_0\langle \vec{p}_1\, \vec{p}_2 \cdots | T\Big(e^{-i\int_{-T}^{T} dt\, H_I}\Big) \underbrace{| \vec{k}_A\, \vec{k}_B \rangle_0}$$

$$\boxed{\langle \vec{p}_1\, \vec{p}_2 \cdots | i\underline{\underline{\mathbb{T}}} | \vec{k}_A\, \vec{k}_B \rangle = \lim_{T \to \infty} \Big({}_0\langle \vec{p}_1\, \vec{p}_2 \cdots | \underbrace{T\Big(e^{-i\int_{-T}^{T} dt\, H_I}\Big)} | \vec{k}_A\, \vec{k}_B \rangle_0 \Big)}$$

no bubble + no noninteracting

↑

connected

+

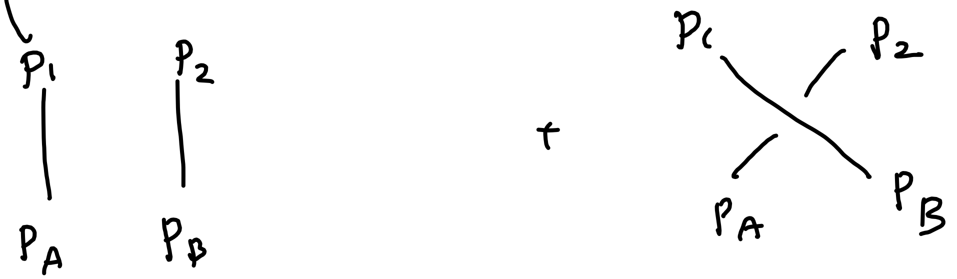
amputated

$2 \longrightarrow 2$

$${}_0\langle \vec{p}_1 \vec{p}_2 \underset{\uparrow}{|} \vec{p}_A \vec{p}_B \rangle_0 = \sqrt{2E_1 2E_2 2E_A 2E_B}\ \underbrace{\langle 0|\, a_1 a_2 a_A^{\dagger} a_B^{\dagger} |0\rangle}$$

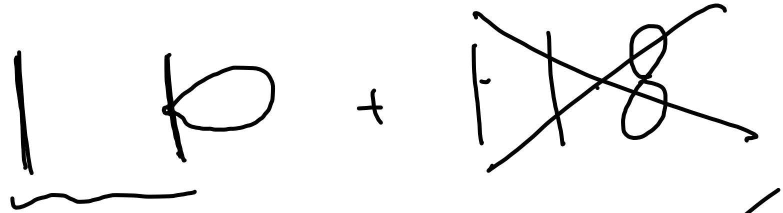
$$= \sqrt{\quad ''\quad}\ (2\pi)^6 \left(\delta(\vec{p}_1 - \vec{p}_A)\, \delta(\vec{p}_2 - \vec{p}_B) + \delta(\vec{p}_1 - \vec{p}_B)\, \delta(\vec{p}_2 - \vec{p}_A) \right)$$

(cf) $\left(|\vec{p}\rangle = \sqrt{2E_p}\, a_{\vec{p}}^{\dagger} |0\rangle \right)$

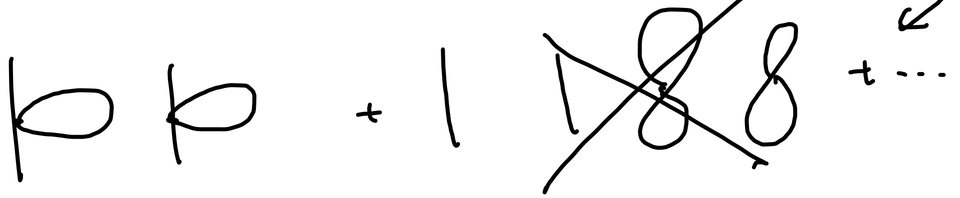


non-interacting terms

λ; 1st order



λ^2 2nd order



λ order : $${}_0\langle \vec{p}_1, \vec{p}_2 | T\left(-\frac{i\lambda}{4!}\int d^4x\, \phi^4\right) | \vec{p}_A\, \vec{p}_B\rangle_0$$

Wick theorem $$= \underline{{}_0\langle \vec{p}_1, \vec{p}_2 | N\left(-\frac{i\lambda}{4!}\int d^4x\, \phi^4 + \text{contractions}\right) | \vec{p}_A, \vec{p}_B\rangle_0}$$

(cf) Correlation function

$$\underline{\underline{\langle 0}} | \underbrace{N\left(\phi(x)\phi(y)\left(-\frac{i\lambda}{4!}\right)\int d^4x\, \phi^4 + \text{contractions}\right)}_{\text{fully contracted terms only}} \underline{\underline{|0\rangle}}$$

$$\phi^+(x)|\vec{p}\rangle_0 = \int \frac{d^3k}{\cancel{(2\pi)^3}} \frac{1}{\cancel{\sqrt{2E_k}}}\, a_{\vec{k}}\, e^{-ik\cdot x}\, \cancel{\sqrt{2E_{\vec{p}}}}\, a^{\dagger}_{\vec{p}}|0\rangle = e^{-ip\cdot x}|0\rangle$$

$$\cancel{(2\pi)^3}\,\delta^{(3)}(\vec{k}-\vec{p}) \qquad k^0 = p^0$$

↓

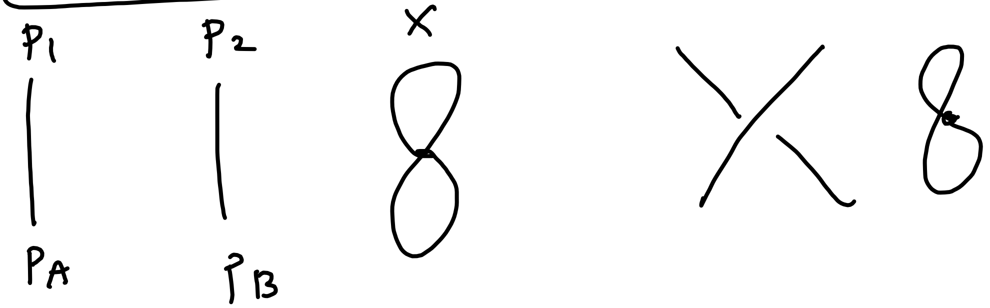
$$\overline{\phi(x)|\vec{p}}\rangle = e^{-ip\cdot x}$$

$$\langle\vec{p}|\phi^-(x) = e^{ip\cdot x}\langle 0| \to \langle\overline{\vec{p}|\phi}(x) = e^{ip\cdot x}$$

$$\uparrow \langle 0| a_{\vec{p}}\, a^{\dagger}_{\vec{k}}\, e^{ik\cdot x}$$

(ex)

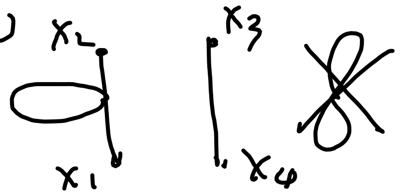
$$ {}_0\langle \vec{p}_1, \vec{p}_2 | N\left(-\frac{i\lambda}{4!} \int d^4x \; \phi\phi\phi\phi \right) | \vec{p}_A, \vec{p}_B \rangle_0 $$

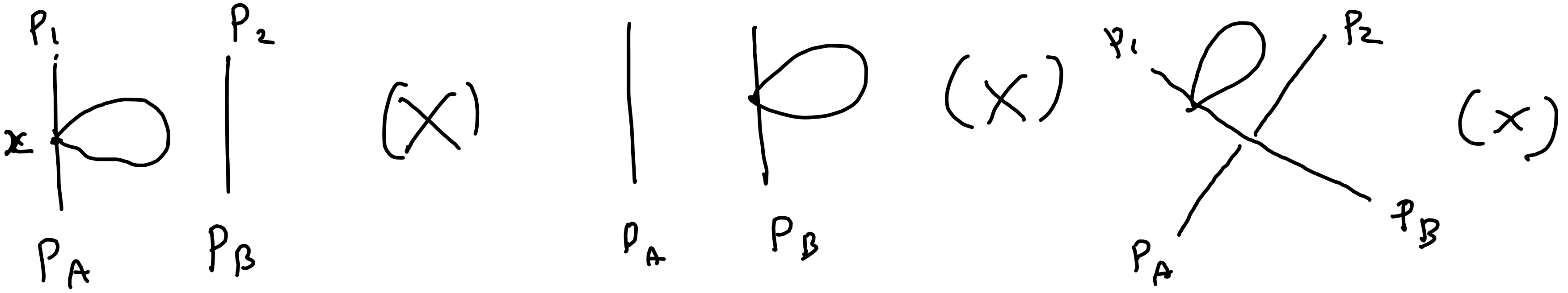


$$ {}_0\langle \vec{p}_1, \vec{p}_2 | N\left(-\frac{i\lambda}{4!} \int d^4x \; \phi\phi\phi\phi \right) | \vec{p}_A, \vec{p}_B \rangle_0 $$

x

(cf) Corr. fnct





$\Rightarrow$ no non-interacting terms $\equiv$ all lines are connected.

$e^{ip_1\cdot x}$ $\qquad\qquad$ $e^{-ip_B\cdot x}$

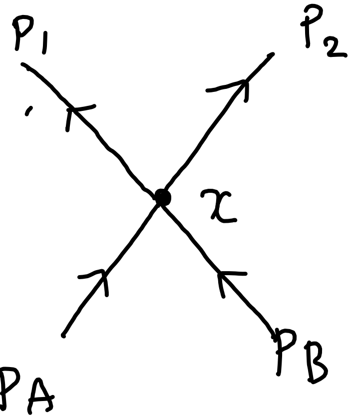
$${}_0\langle \vec{p}_1, \vec{p}_2 |\, N\left(-\frac{i\lambda}{4!}\underbrace{\int d^4x}\ \phi\,\phi\,\phi\,\phi\right) | \vec{p}_A, \vec{p}_B\rangle_0$$

$e^{ip_2\cdot x}$ $\qquad\qquad$ $e^{-ip_A\cdot x}$

$$\int d^4x\ e^{i(p_1+p_2-p_A-p_B)\cdot x}$$

$$\parallel$$

$$(2\pi)^4\delta^{(4)}(p_1+p_2-p_A-p_B)$$



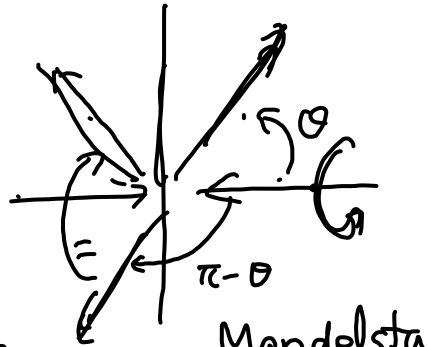
$$= -\frac{i\lambda}{\cancel{4!}}\ 4\cdot 3\cdot 2\cdot 1 = \underline{(-i\lambda)}\ \underbrace{\overset{(2\pi)^4}{\delta^{(4)}}(p_A+p_B-p_1-p_2)}$$

$$\equiv iM\ (2\pi)^4\delta^{(4)}(\Sigma p)$$

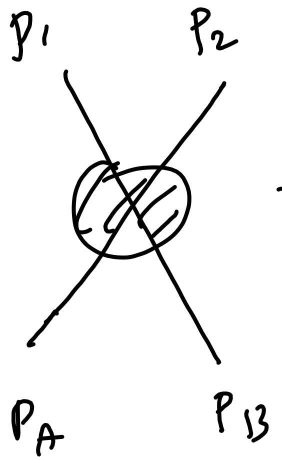
$$\therefore \boxed{M = -\lambda}$$

$$\therefore \left(\frac{d\sigma}{d\Omega}\right)_{CM,0} = \frac{\lambda^2}{64\pi^2 E_{cm}^2}$$ ← Ω-indep.

$$\underline{\sigma = \frac{1}{2}\cdot\frac{\lambda^2}{16\pi E_{cm}^2} = \frac{\lambda^2}{32\pi}\frac{1}{s}}$$



CM: $\vec{p}_A+\vec{p}_B=0$ $\quad$ $p_A+p_B=(\overset{E_A+E_B}{\overset{\parallel}{E_{cm}}},\vec{0})$ $\quad$ $(p_A+p_B)^2\equiv s$ ← Mandelstam variable



$$s = (p_A + p_B)^2 = (p_1 + p_2)^2$$

$$t = (p_A - p_1)^2$$

$$u = (p_A - p_2)^2$$

s, t, u : Mandelstam

$$\left(s + t + u = 4m^2 \right)$$

$$\frac{1}{2!}\,{}_0\langle \overset{e^{ip_1\cdot x}}{\vec{p}_1},\underset{e^{ip_2\cdot x}}{\vec{p}_2}\,|\,N\left(\left(-\frac{i\lambda}{4!}\right)^2\int d^4x\;\phi\,\phi\,\phi\,\phi\int d^4y\;\phi\,\phi\,\phi\,\phi\right)|\,\overset{e^{-ip_A\cdot x}}{\vec{p}_A},\underset{e^{-ip_B\cdot y}}{\vec{p}_B}\rangle_0$$

$$D_F(x-y)\;\downarrow\;\int\frac{d^4p'}{(2\pi)^4}\frac{i}{p'^2-m^2}\,e^{-i(x-y)\cdot p'}$$

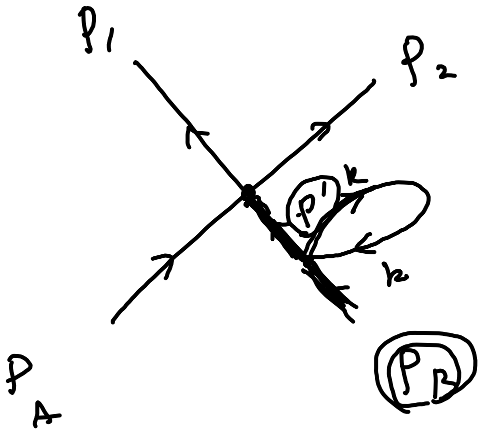
$$D_F(y-y)\longrightarrow\underbrace{\int\frac{d^4k}{(2\pi)^4}}\frac{i}{k^2-m^2}$$

$$=\frac{1}{2}(-i\lambda)^2\underbrace{\int d^4x\;e^{i(p_1+p_2-p_A-p')\cdot x}}_{(2\pi)^4\delta^{(4)}(p_1+p_2-p_A-p')}\underbrace{\int d^4y\;e^{-i(p_B-p')\cdot y}}_{(2\pi)^4\delta^{(4)}(p_B-p'+k-k)}$$

$$\underbrace{\int\frac{d^4p'}{\cancel{(2\pi)^4}}\frac{\cancel{(2\pi)^4}\,i\,\delta^{(4)}(p_B-p')}{p'^2-m^2}}\cdot\underbrace{\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2}}_{\infty}$$

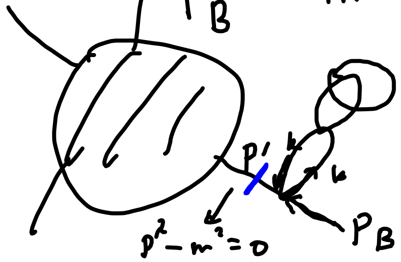
$$\downarrow$$

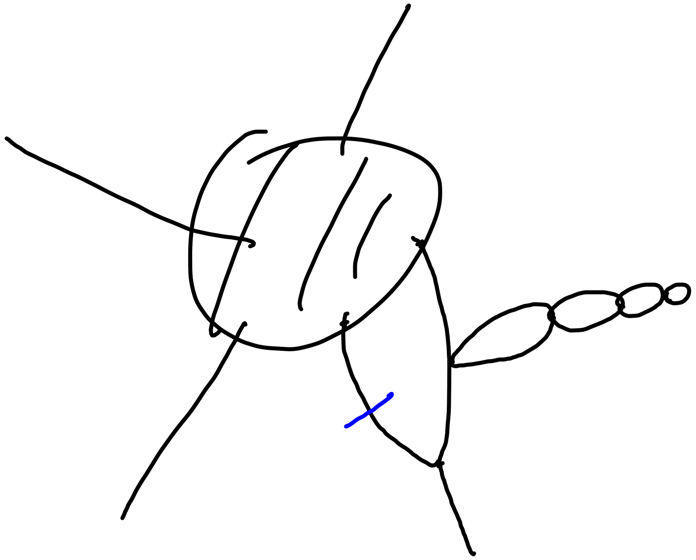
$$\frac{i}{p_B^2-m^2}=\frac{i}{0}=\infty$$



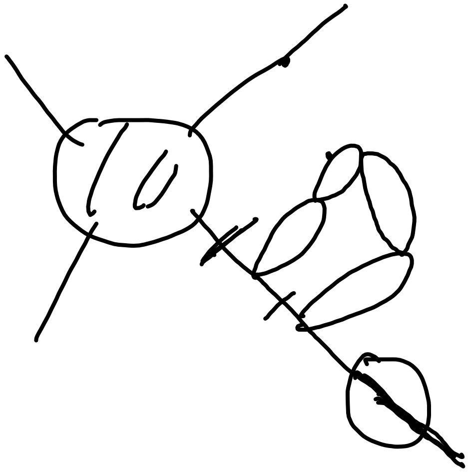
$$\underset{\sim\sim\sim}{p_B^2=m^2}$$

"amputated"





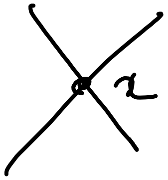
non amputated ✓



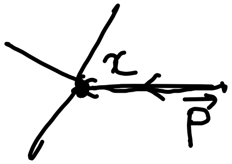
amputated

Real space

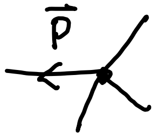
$D_F(x-y)$



$(-i\lambda)\int d^4x$



in-state $e^{-ip\cdot x}$

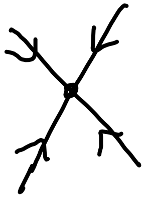


out-state $e^{ip\cdot x}$

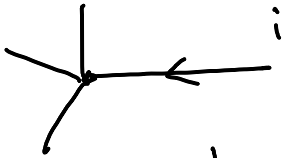
$i\mathcal{M}\,(2\pi)^4\,\delta^{(4)}\left(\sum p_{in}-\sum p_{out}\right)$

Momentum space

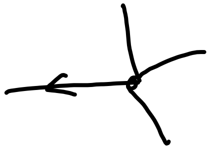
$= \frac{i}{p^2-m^2+i\epsilon}$



$= -i\lambda$



in-state $= 1$



out $= 1$

$i\mathcal{M}$

$i\mathcal{M}$ = sum of all connected, amputated diagrams

4.7. Feynman rules for fermions

$$T \rightarrow N$$

$$T(\psi(x)\bar{\psi}(y)) = \begin{cases} \psi(x)\bar{\psi}(y) & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x) & y^0 > x^0 \end{cases}$$

$$\psi(x) = \underbrace{\int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_{s=1,2} \Big(a^s_{\vec{p}} u^s(\vec{p}) e^{-ip\cdot x}} + b^{s\dagger}_{\vec{p}} v^s(\vec{p}) e^{ip\cdot x} \Big)$$

$$= \psi^+ + \psi^-$$

$$N(\psi(x)\bar{\psi}(y)) = N\Big(\big(\underline{\overset{a}{\psi^+}}(x) + \overset{b^\dagger}{\psi^-}(x)\big)\big(\overset{b}{\bar{\psi}^+}(y) + \underline{\overset{a^\dagger}{\bar{\psi}^-}}(y)\big)\Big)$$

$$= \psi^+\bar{\psi}^+ + \psi^-\bar{\psi}^+ + \psi^-\bar{\psi}^- - \bar{\psi}^-(y)\psi^+(x)$$

$$T(\psi(x_1)\cdots\cdots) = N(\psi(x_1)\cdots \quad + \text{all contractions} \quad)$$

$$\uparrow$$

$$S_F(x-y) = \{\psi(x), \bar{\psi}(y)\} = \overbracket{\psi\bar{\psi}}$$

Yukawa theory

$$H = H_{\text{Dirac}}^{(\psi,\bar{\psi})} + H_{\text{K-G}}^{(\phi)} + \int d^3x\; g\,\bar{\psi}\psi\phi$$

$$\Rightarrow \quad e^{-ig\int d^4x\;\bar{\psi}\psi\phi}$$

$a^\dagger|0\rangle$

(ex)
$${}_0\langle \overbrace{\vec{p}'}^{\text{fermion}}, \vec{k}' |\, T\Big(e^{-ig\int d^4x\;\bar{\psi}\psi\phi} \Big) \,|\, \overbrace{\vec{p}}^{\text{fermion}}, \vec{k} \rangle_0$$

leading

$$= \frac{1}{2!}(-ig)^2\; {}_0\langle \overbrace{\vec{p}'}^{\text{fermion}}, \vec{k}' |\, T\Big(\int d^4x\; \bar{\psi}\psi\phi \int d^4y\; \bar{\psi}\psi\phi \Big) \,|\, \overbrace{\vec{p}}^{\text{fermion}}, \vec{k} \rangle_0$$

$$\overbrace{\psi(y)\;|\vec{p}\rangle_0} = \int \frac{d^3p'}{(2\pi)^3}\frac{1}{\cancel{\sqrt{2E_{\vec{p}'}}}}\sum_{s'=1,2} \overbrace{a^{s'}_{\vec{p}'}\, u^{s'}(\vec{p}')\, e^{-ip'\cdot x}\; \cancel{\sqrt{2E_{\vec{p}}}}\; a^\dagger_{\vec{p},s}}^{(2\pi)^3\,\delta^{ss'}\,\delta^{(3)}(\vec{p}'-\vec{p})}|0\rangle$$

$$\underbrace{\sqrt{2E_{\vec{p}}}\; a^\dagger_{\vec{p},s}|0\rangle}_{=|\vec{p},s\rangle_0}$$

$$\overbrace{\psi(y)\;|\vec{p}\rangle_0} = u^s(\vec{p})\, e^{-ip\cdot x}\,|0\rangle$$

$$\overbrace{\overline{\psi}(y)\,|\vec{p}\rangle_0} = 0$$

feri

$$\downarrow$$

$$\overline{\psi}^{\dagger}$$

$$\psi \sim a + b^{\dagger}$$

$$\overline{\psi} \sim a^{\dagger} + b$$

antifermion

$$\underbrace{\overline{\psi}(x)}\;\underbrace{|\vec{p}\rangle_0}_{\sqrt{2E_{\vec{p}}}\,b^{\dagger}_{\vec{p}}|0\rangle}$$

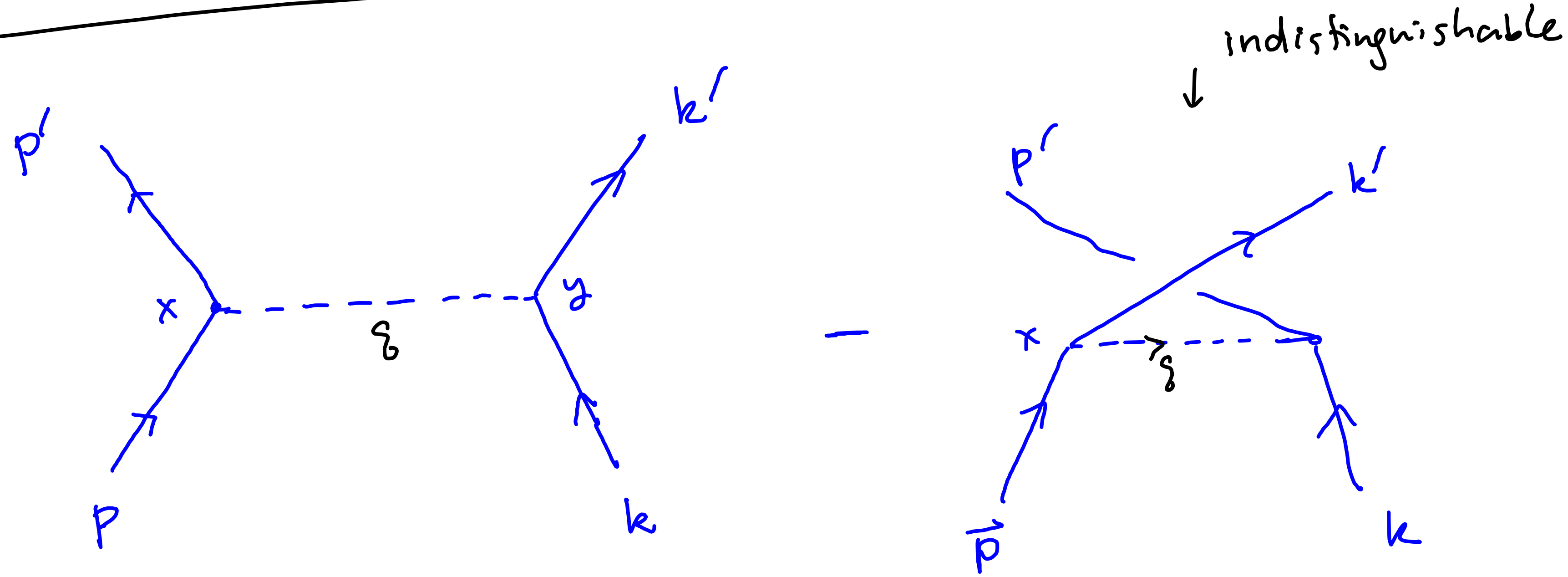
$$\parallel$$

$$\overline{\psi}^{\dagger}(x) = \int \frac{d^3p'}{\cancel{(2\pi)^3}}\,\frac{1}{\cancel{\sqrt{2E_{\vec{p}'}}}}\sum_{s'=1,2}\left(\underset{\sim}{b^{s'}_{\vec{p}'}}\,\overline{v}^{s'}(\vec{p}')\,e^{-ip'\cdot x}\right)\cancel{\sqrt{2E_{\vec{p}}}}\;\underset{\sim}{b^{\dagger}_{\vec{p}}}\,|0\rangle$$

$$\overline{v}^{s}(\vec{p})\;e^{-ip\cdot x}$$

$$\boxed{\begin{aligned}\overbrace{\psi(x)\,|\vec{p},s,\text{fermion}\rangle} &= u^{s}(p)\,e^{-ip\cdot x}\\ \overbrace{\overline{\psi}(x)\,|\vec{p},s,\text{anti-fermion}\rangle} &= \overline{v}^{s}(p)\,e^{-ip\cdot x}\end{aligned}}$$

$$\boxed{\begin{aligned} \langle \vec{p}, s, \text{fermion} |\ \bar{\psi}(x) &= \bar{u}^{s}(p)\, e^{ip\cdot x} \\ \langle \vec{p}, s, \text{anti-fermion} |\ \psi(x) &= v^{s}(p)\, e^{ip\cdot x} \end{aligned}}$$



$$i\mathcal{M} = \frac{1}{2!}(-ig)^2 \left(\underset{\sim\sim}{\overset{\uparrow}{\bar{u}(p')}}\ \underset{\sim}{u(p)}\ \frac{i}{q^2-m^2}\Bigg|_{q=p'-p} \bar{u}(k')\, u(k) - \bar{u}(p')\, u(k)\ \frac{i}{q^2-m^2}\Bigg|_{p-k'} \bar{u}(k')u(p) \right)$$

$$\Longrightarrow \quad |\mathcal{M}|^2$$

$$\overbrace{\psi(x)\;|\underbrace{\vec p, s, \text{fermion}}_{\propto\, a^{+s}_{\vec p}|0\rangle}}^{}\rangle = u^s(p)\, e^{-ip\cdot x}$$

$a + b^+ \nearrow$

$$\overbrace{\bar\psi(x)\;|\underbrace{\vec p, s, \text{anti-fermion}}_{\propto\, b^+|0\rangle}}^{}\rangle = \bar v^s(p)\, e^{ip\cdot x}$$

$a^+ + b \nearrow$

$$\overbrace{\bar\psi \quad |\vec p, s\rangle_f}^{x} \qquad\qquad \overbrace{\psi\; |\vec p, s\rangle_{\bar f}}^{x}$$

$\swarrow a^+ + b$

$$\overbrace{\langle \vec p, s, \text{fermion}|\;\bar\psi}^{}(x) = \bar u^s(p)\, e^{ip\cdot x}$$

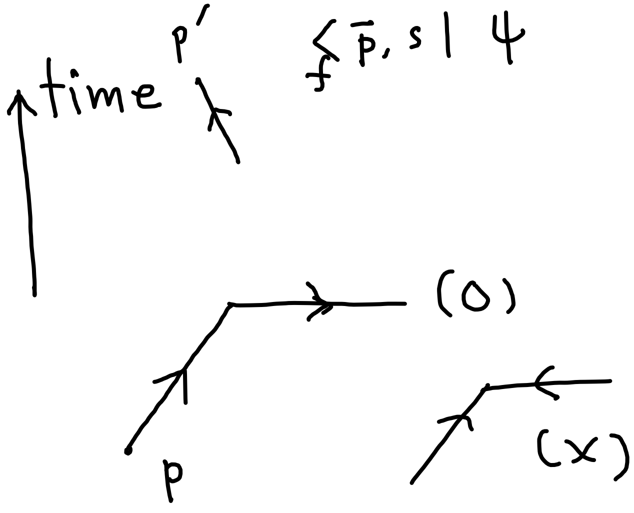
${}_f\langle 0|\, a$ $\qquad$ $a + b^+$

$$\langle \vec p, s, \text{anti-}\overbrace{\text{fermion}|\;\psi}^{}(x) = v^s(p)\, e^{ip\cdot x}$$

${}_{\bar f}\langle 0|\, b$

$$\overbrace{{}_f\langle \vec p, s|\;\psi}^{x} \qquad\qquad \overbrace{{}_{\bar f}\langle \vec p, s|\;\bar\psi}^{x}$$

fermion:



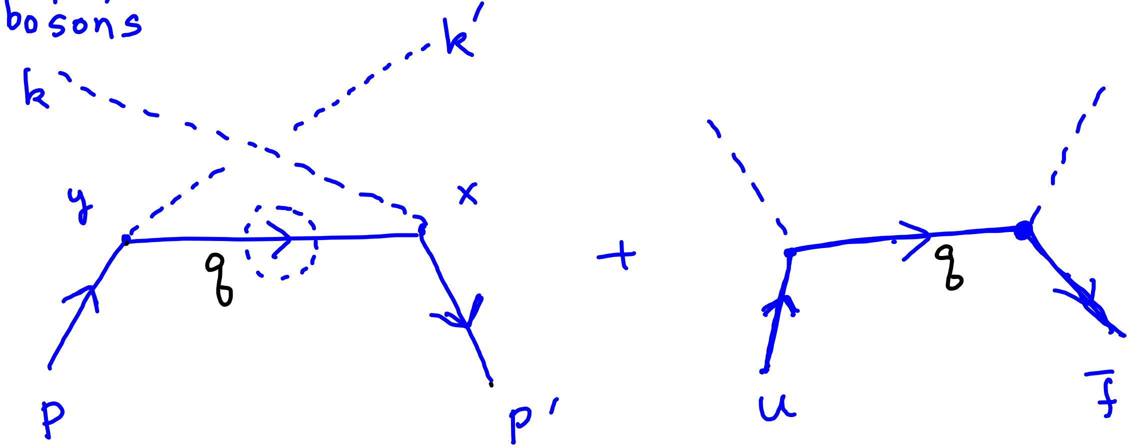
anti fermion:

$$\frac{1}{2!}(-ig)^2\,{}_0\!\left\langle \vec{k}, \vec{k}' \right| T\left(\int d^4x\, \bar\psi\psi\phi \int d^4y\, \bar\psi\psi\phi\right)\left| \vec{p}, \vec{p}'\right\rangle_0$$

bosons

f $\bar f$

$\langle p|\phi(x) = e^{ip\cdot x}$



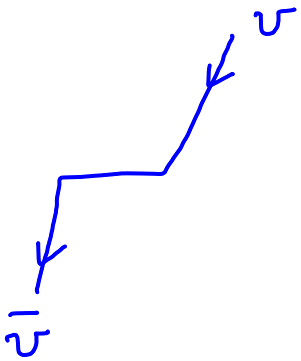
$\bar v(\cdots\cdots)u$

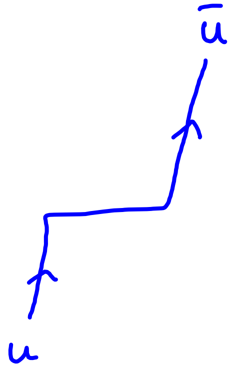
$e^{ik\cdot x}\cdot e^{ik'\cdot y}$

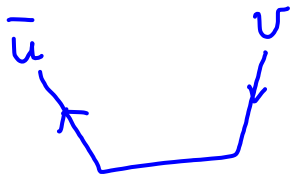
$$\frac{1}{2!}(-ig)^2\int d^4x \int d^4y\; \bar v(p')\, e^{-ip'\cdot x}\int \frac{d^4q}{(2\pi)^4}\,\frac{i(\not q+m)}{q^2-m^2}\, e^{-iq\cdot(x-y)}\; u(p)\, e^{-ip\cdot y}$$

$$= \frac{1}{2!}(-ig)^2\int \frac{d^4q}{(2\pi)^4}\;\overbrace{\int d^4x\; e^{-ix\cdot(p'+q-k)}}^{(2\pi)^4\delta^{(4)}(p'+q-k)}\;\overbrace{\int d^4y\; e^{-iy\cdot(p-q-k')}}^{(2\pi)^4\delta^{(4)}(p-q-k')}\;\overbrace{\bar v(p')\,\frac{i(\not q+m)}{q^2-m^2}\,u(p)}^{\text{scalar}}$$

$$(\cdot\;\cdot\;\cdot\;\cdot)\begin{pmatrix}\cdot&\cdot&\cdot&\cdot\\ \cdot&\cdot&\cdot&\cdot\\ \cdot&\cdot&\cdot&\cdot\\ \cdot&\cdot&\cdot&\cdot\end{pmatrix}\begin{pmatrix}\cdot\\ \cdot\\ \cdot\\ \cdot\end{pmatrix}$$







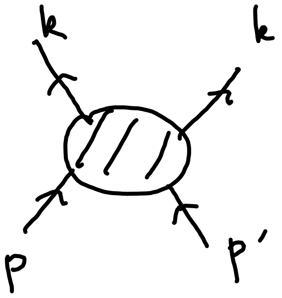
$\bar u(\cdots\cdot)v$

$$= \frac{1}{2!}(-ig)^2 \int \frac{d^4q}{\cancel{(2\pi)^4}} \cancel{(2\pi)^4} \delta^{(4)}(\underset{\substack{\downarrow \\ q = k - p'}}{p' + q - k}) \quad (2\pi)^4 \delta^{(4)}(p - q - k') \quad \bar{u}(p') \frac{i(\not{q}+m)}{q^2 - m^2} u(p)$$

$$+ \quad \frac{1}{2!}(-ig)^2 \int \frac{d^4q}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(p' + q - k') \quad (2\pi)^4 \delta^{(4)}(p - q - k) \quad \bar{u}(p') \frac{i(\not{q}+m)}{q^2 - m^2} u(p)$$

$$= \frac{1}{2!}(-ig)^2 \quad (2\pi)^4 \delta^{(4)}(p + p' - k - k') \quad \bar{u}(p') \left.\frac{i(\not{q}+m)}{q^2 - m^2}\right|_{q = k - p'} u(p)$$

$$+ \quad \frac{1}{2!}(-ig)^2 \quad \underbrace{(2\pi)^4 \delta^{(4)}(p + p' - k - k')} \quad \bar{u}(p') \left.\frac{i(\not{q}+m)}{q^2 - m^2}\right|_{q = k' - p'} u(p)$$



$$|\vec{p},\vec{k}\rangle_{ff} \propto a^{\dagger}_{\vec{p}}\, a^{\dagger}_{\vec{k}}|0\rangle$$

$$\dagger \downarrow \qquad\qquad \downarrow \dagger$$

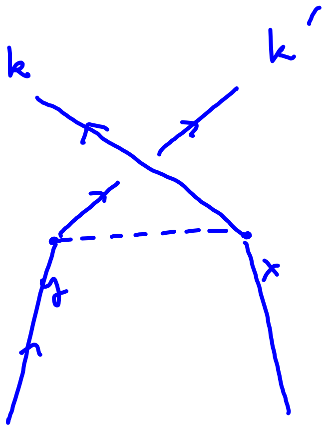
$$_{ff}\langle \vec{p}',\vec{k}'| \qquad \underline{\langle 0|a_{\vec{k}}\,a_{\vec{p}}} = -a_{\vec{p}}\,a_{\vec{k}}$$

$$\langle \vec{p}',\vec{k}'|\vec{p},\vec{k}\rangle \propto \delta^{(4)}(p'-p)\,\delta^{(4)}(k'-k)$$

$$_{ff}\langle \vec{p}',\vec{k}'|(\bar{\psi}\psi)_x(\bar{\psi}\psi)_y|\vec{p},\vec{k}\rangle_{ff}$$

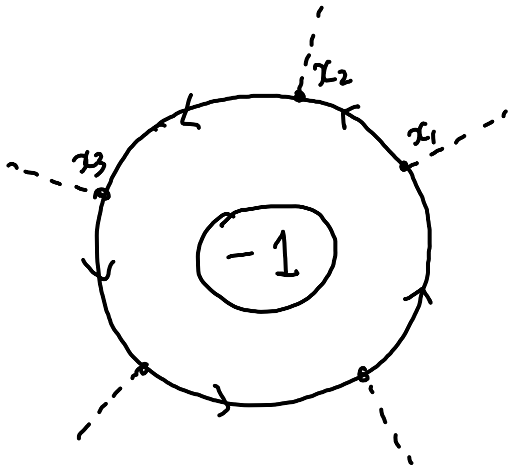
$$\langle 0|a_{\vec{k}'}\,a_{\vec{p}'}(\bar{\psi}\psi)_x(\bar{\psi}\psi)_y\,a^{\dagger}_{\vec{p}}\,a^{\dagger}_{\vec{k}}|0\rangle$$

$$(-1)$$



(-1)

fermion loop



$$(\bar\psi\psi)_{x_1}(\bar\psi\psi)_{x_2}\cdots(\bar\psi\psi)_{x_n}$$

$$= \underline{\underline{(-1)}}\,S_F(x_1-x_2)\,S_F(x_2-x_3)\cdots S_F(x_n-x_1)$$

Nonrelativistic limit: $|\vec p| \ll m$; $p = (\underset{\sqrt{\vec p^2 + m^2}}{E_{\vec p}}, \vec p) \approx (m, \vec p) \approx (m, 0)$, $p' \simeq (m, \vec p')$

& distinguishable

$$i\mathcal{M} = \frac{1}{2}(-ig)^2 \left(\bar u(p')\, u(p) \frac{i}{q^2 - m_\phi^2}\Bigg|_{q = p'-p} \bar u(k')\, u(k) - \xcancel{\bar u(p')\, u(k) \frac{i}{q^2 - m_\phi^2}\Bigg|_{p-k'} \bar u(k')\, u(p)} \right)$$

$$q = p' - p \cong (0, \vec p' - \vec p) \qquad q^2 \approx -(\vec p' - \vec p)^2$$

$$p\cdot\sigma = m\mathbb{1} - \vec p\cdot\vec\sigma \approx m.$$

$$u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\ \xi^s \\ \sqrt{p\cdot\bar\sigma}\ \xi^s \end{pmatrix} \approx \sqrt{m}\begin{pmatrix} \xi^s \\ \xi^s \end{pmatrix}$$

$$v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\ \eta^s \\ -\sqrt{p\cdot\bar\sigma}\ \eta^s \end{pmatrix} \approx \sqrt{m}\begin{pmatrix} \eta^s \\ -\eta^s \end{pmatrix}$$

Born Approx

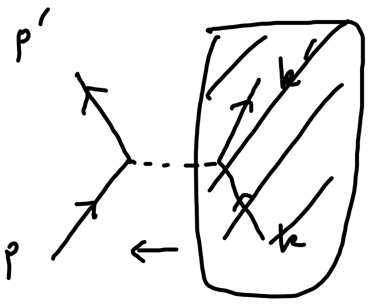
$$\int V(\vec x)\, e^{-i\vec x\cdot\overbrace{(\vec p' - \vec p)}^{\vec q}} d^3\vec x \approx \tilde V(\vec q)$$

$$\bar u^{s'}(p')\, u^s(p) \approx m\, (\xi^{s'\dagger}\ \ \xi^{s'\dagger}) \overbrace{\begin{pmatrix} & 1 \\ 1 & \end{pmatrix}}^{\gamma^0} \begin{pmatrix} \xi^s \\ \xi^s \end{pmatrix} = 2m \underbrace{\xi^{s'\dagger}\xi^s}_{\delta_{ss'}} = 2m\,\delta_{ss'}$$

$$\bar u^{r'}(k')\, u^r(k) \approx 2m\,\delta_{rr'}$$

$$i\mathcal{M} = \frac{\overbrace{(ig)^2\ (-i)}^{+ig^2}\ (2m)^2\ \delta_{ss'}\,\delta_{rr'}}{(\underbrace{\vec p' - \vec p}_{\vec q})^2 + m_\phi^2}$$

(cf)



Q.M.

$= \langle \vec p' | iT | \vec p \rangle$

Born Approx.

$= -i\,\tilde V(\vec q)\,(2\pi)\,\delta(E_p - E_p)$

$$\tilde{V}(\vec{q}) = \frac{-g^2}{|\vec{q}^2| + m_\phi^2}$$

↓

$$V(\vec{x}) = \int \frac{d^3q}{(2\pi)^3} \frac{-g^2}{|\vec{q}|^2 + m_\phi^2} e^{i\vec{q}\cdot\vec{x}} = \frac{-g^2}{(2\pi)^2} \int_0^\infty dq \underbrace{\frac{1}{q^2+m_\phi^2}} \underbrace{\int_{-1}^{1} dt\, e^{iq|\vec{x}|t}}_{\frac{1}{iq|\vec{x}|}\left(e^{iq|\vec{x}|} - e^{-iq|\vec{x}|}\right)}$$

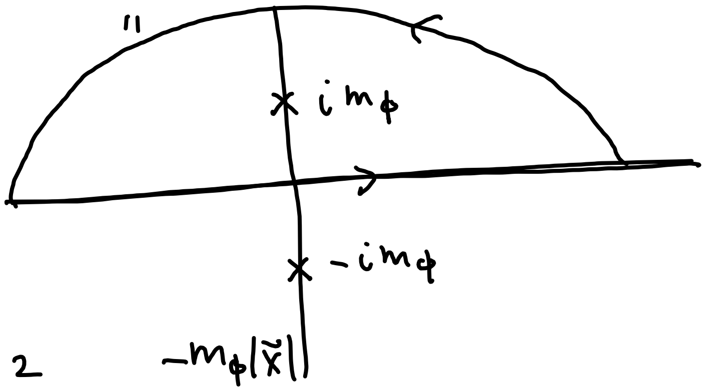
$q = |\vec{q}|$

$$d^3q = q^2\, dq\, d\underbrace{\cos\theta}_{t}\, d\phi$$

$$-\int_0^{-\infty} e^{iq|\vec{x}|} = \int_{-\infty}^{0}$$

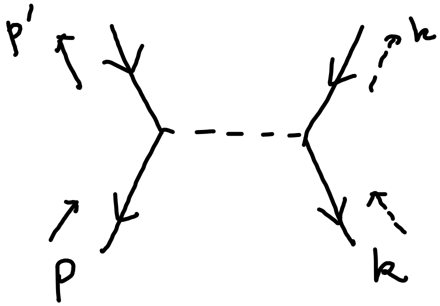
$$= -\frac{g^2}{(2\pi)^2} \frac{1}{i|\vec{x}|} \int_{-\infty}^{\infty} dq \frac{e^{\overbrace{iq|\vec{x}|}}\, q}{i(q^2+m_\phi^2)} \leftarrow \frac{q}{\underbrace{(q-im_\phi)(q+im_\phi)}}$$

$$\to 2\pi i \frac{i\cancel{m_\phi}}{2i\cancel{m_\phi}} e^{-m_\phi|\vec{x}|}$$



attractive
↓
$$= -\frac{g^2}{4\pi} \frac{e^{-m_\phi|\vec{x}|}}{|\vec{x}|}$$

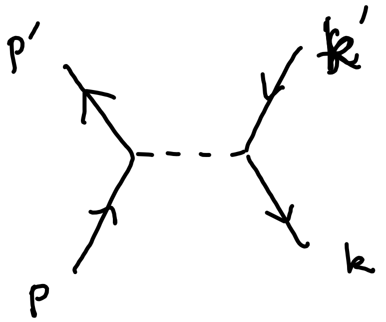
$\bar{f}\,\bar{f} \to \bar{f}\,\bar{f}$



$$\underbrace{\bar{v}^{s'}(p)\, v^{s}(p')}_{m\,(\eta^{s'\dagger}, -\eta^{s'\dagger})\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}\eta^{s}\\-\eta^{s}\end{pmatrix} = -2m\,\delta^{s's}} \quad \underbrace{\bar{v}^{r'}(k)\, v^{r}(k')}_{-2m\,\delta^{r'r}} \quad \Big\}\ (-1)^2 \ ✓$$

attractive

$f\,\bar{f} \to f\,\bar{f}$



$$\underbrace{\underbrace{\bar{v}^{s'}(p)\, v^{s}(p')}_{-2m\,\delta^{s's}} \quad \underbrace{\bar{u}^{r'}(k')\, u^{r}(k)}_{2m\,\delta^{rr'}}}$$

$$(-1) \times (-1) = (-1)^2$$

attractive

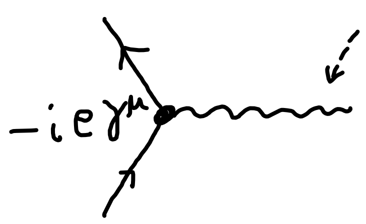
$$\langle p'_f, k'_{\bar{f}} |\, \bar{\psi}\, \psi \quad \bar{\psi}\, \psi \quad | \vec{p}_f, \vec{k}_{\bar{f}} \rangle$$

$$-1\ \langle 0 |\, \underbrace{b_{k'}\, a_{p'}}_{-1}\, \bar{\psi}\, \psi \quad \bar{\psi}\, \psi$$

4.8. QED

$$H_{int} = \int d^3\vec{x}\ \underset{\sim}{e}\ \bar{\Psi}\ \underline{\underline{\gamma^{\mu}}}\ \Psi\ \underline{\underline{A_{\mu}}}$$

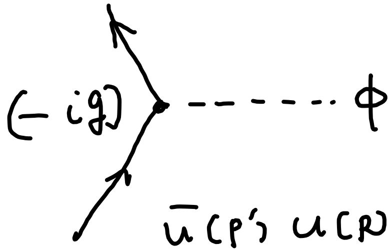
$$e^{\underset{\sim}{-i}\int}$$



$\uparrow$

$\bar{u}\ \gamma^{\mu}\ u$

(cf)



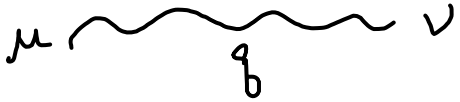
gauge boson ; $A_{\mu = \underset{\sim}{0,1,2,3}} \rightarrow$ not all independent

ghost field

$\Uparrow$

quantized Lagrange

비정게수 field

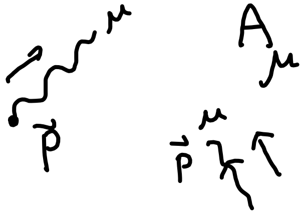


$$-g_{\mu\nu}\ \frac{i}{q^2 + i\varepsilon}$$

if not ~~0 longitudinal polarised~~ } massive only

$$\begin{array}{ccccc} & 0 & 1 & 2 & 3 \\ \varepsilon_{\mu}(|\vec{p}|\hat{z}) = (& 0 & ▨ & ▨ & 0 \ ) \end{array}$$

transversal polarized.



$$\overline{A_{\mu}\ |\vec{p}}\rangle_{\gamma} = \varepsilon_{\mu}(\vec{p})$$

$$_{\gamma}\langle\overline{\vec{p}\,|\ A_{\mu}} = \varepsilon^{*}_{\mu}(\vec{p})$$

$\varepsilon = (0, \vec{\varepsilon})$ $\qquad$ $\vec{\varepsilon} \cdot \vec{p} = 0$

$\vec{p} \propto \hat{z}$ $\qquad$ $\varepsilon = (0, 1, \pm i, 0) \frac{1}{\sqrt{2}} \begin{cases} +: \text{right handed polari} \\ -: \text{left} \quad " \end{cases}$

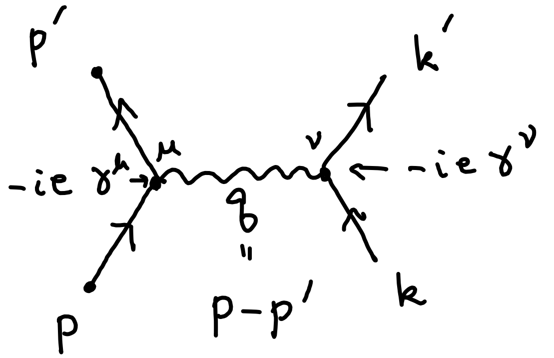
$$\langle 0 | T\{ A_\mu(x) A_\nu(0) \} | 0 \rangle = \int \frac{d^4 q}{(2\pi)^4} \frac{-i g_{\mu\nu}}{q^2 + i\varepsilon} e^{-i q \cdot x}$$

$\mu = \nu = 0$

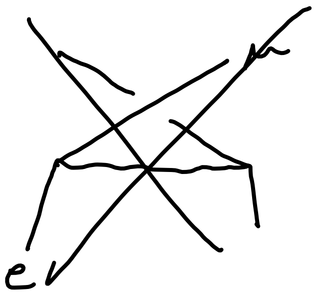
$$\langle 0 | T\{ A_0(\underset{\underset{0}{\uparrow}}{x}) A_0(0) \} | 0 \rangle = \int \frac{d^4 q}{(2\pi)^4} \frac{-i}{\underset{\uparrow\, q_0^2 - |\vec{q}|^2}{q^2 + i\varepsilon}} e^{-i q \cdot x}$$

$$\boxed{\| A_0(0) | 0 \rangle \|^2 < 0} = \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{-1}{2|\vec{q}|} \underbrace{e^{-i q \cdot x}}_{1} \Big|_{q^0 = |\vec{q}|}$$

$ff \to ff$



ee



$p-p' \cong (0, \vec p - \vec p')$

e $\qquad$ μ

$$i\mathcal{M} = (-ie)^2 \underbrace{\bar u(p')\gamma^\mu u(p)}_{ss} \frac{-ig_{\mu\nu}}{q^2+i\varepsilon} \overbrace{\bar u(k')\gamma^\nu u(k)}^{\substack{\nu=0 \text{ dominant} \\ 2m\delta^{rr'}}}$$

non-rel. limit: $\quad \bar u(p')\gamma^0 u(p) = m(\xi^{s\dagger}, \xi^{s\dagger})\gamma^0\gamma^0 \begin{pmatrix}\xi^{s'} \\ \xi^{s'}\end{pmatrix}$

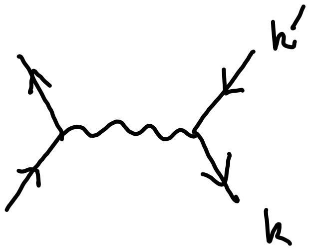
$$= 2m\delta^{ss'}$$

$$= \frac{-ie^2}{|\vec p - \vec p'|^2}(2m\delta^{ss'})(2m\delta^{rr'})$$

$$\Rightarrow V(\vec x) = \frac{e^2}{4\pi}\frac{1}{|\vec x|}$$ Coulomb potential (repulsive)

$f\bar{f} \to f\bar{f}$ $\qquad\qquad$ $\underline{\bar{f}\bar{f} \to \bar{f}\bar{f}}$

repulsive.



$\langle \vec{p}, \vec{k}|$

$\parallel$

$\langle 0| b_{\vec{k}} a_{\vec{p}}$

$\downarrow$

$$i\mathcal{M} = (-ie)^2\, \bar{u}(p')\gamma^\mu u(p)\, \frac{-i g_{\mu\nu}}{q^2 + i\epsilon}\, \underbrace{\bar{v}(k)\gamma^\nu v(k')}_{m\,(\eta^{s\dagger}, -\eta^{s\dagger})\,\underbrace{\gamma^0\gamma^{\overset{0}{\nu}}}_{\mathbb{1}}\begin{pmatrix}\eta^{s'}\\ -\eta^{s'}\end{pmatrix}}\,(-1)$$

$$\underbrace{m\,(\eta^{s\dagger}, -\eta^{s\dagger})\,\gamma^0\gamma^{\overset{0}{\nu}}\begin{pmatrix}\eta^{s'}\\ -\eta^{s'}\end{pmatrix}}_{2\,\delta^{ss'}}$$

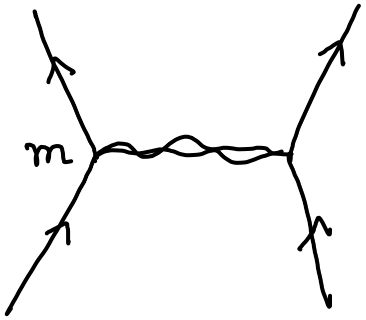
nonrel. $\quad 0 \quad 0$

$$= (-1)\,(ff \to ff)$$ → attractive

If gravity is a guge theory with spin 2 graviton.

$$(\mu\nu) \sim\sim\sim (\rho\sigma)$$

$$= \left((-g_{\mu\rho})(-g_{\nu\sigma}) + (\rho \leftrightarrow \sigma)\right) \overset{g_{\mu\nu}}{=} \frac{i}{q^2+i\epsilon}$$



not good ∵ nonrenorm.

$ff \to ff \quad f\bar{f} \to f\bar{f} \quad \bar{f}\bar{f} \to \bar{f}\bar{f}$

⇒ all attractive

gauge boson	ff, $\bar{f}\bar{f}$	$f\bar{f}$
scalar (Yukawa) - - - - -	attract.	attract.
vector (QED) 〰	repulsive	attract.
* tensor (gravity) ≈	attract	attract