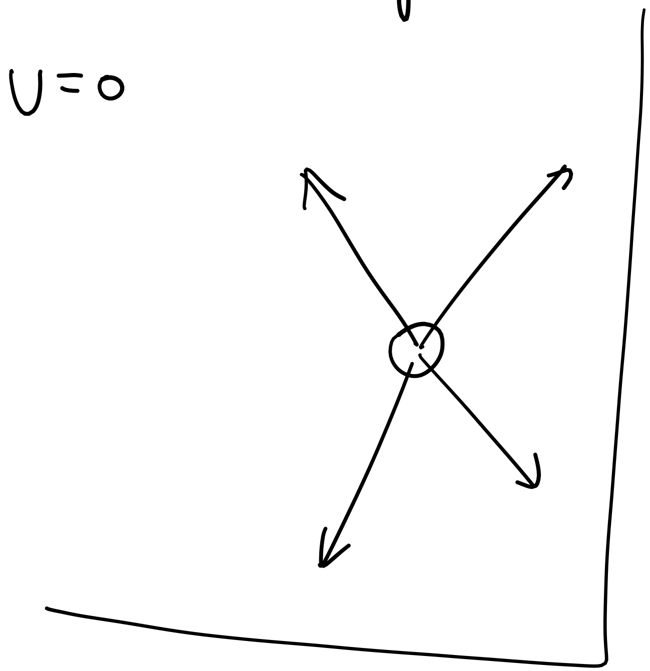
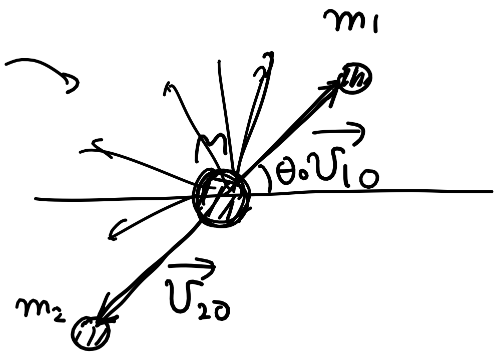
Chap IV. Collision.

1. Disintegration



① 1 → 2 :

(1) CM frame "0"



$M = m_1 + m_2$

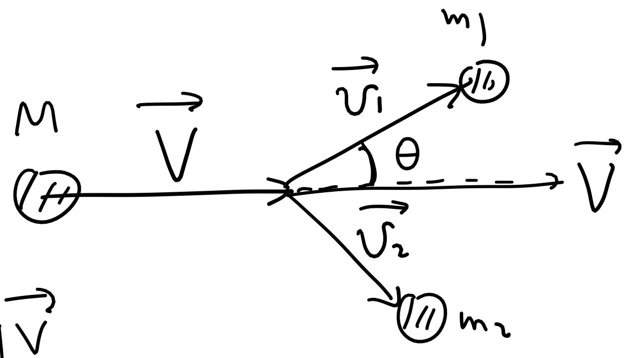
$$\underbrace{m_1 \vec{v}_{10}}_{\equiv \vec{p}_0} + \underbrace{m_2 \vec{v}_{20}}_{-\vec{p}_0} = 0$$

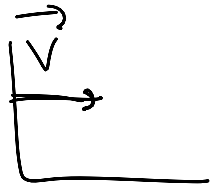
$$E_i = E_{1i} + \frac{1}{2} m_1 v_{10}^2 + E_{2i} + \frac{1}{2} m_2 v_{20}^2$$

$$\epsilon = E_i - E_{1i} - E_{2i} = \frac{1}{2} m_1 \underbrace{v_{10}^2}_{\frac{p_0^2}{m_1^2}} + \frac{1}{2} m_2 \underbrace{v_{20}^2}_{\frac{p_0^2}{m_2^2}} > 0$$

$$= \frac{1}{2} p_0^2 \left(\frac{1}{m_1} + \frac{1}{m_2} \right) = \underbrace{\frac{p_0^2}{2m}}_{} = \frac{m_1^2 v_{10}^2}{2m}$$

(2) LAB

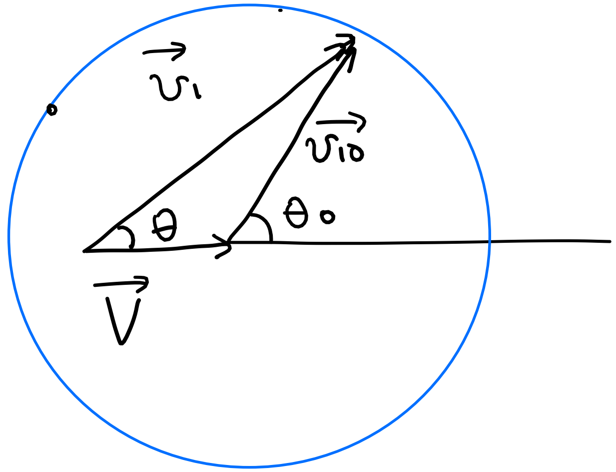




$$\underbrace{\vec{v}_1 - \vec{V} = \vec{v}_{10}} \rightarrow v_1^2 + V^2 - \underbrace{\vec{v}_1 \cdot \vec{V}}_{v_1 V \cos\theta} = v_{10}^2$$

$$\vec{v}_2 - \vec{V} = \vec{v}_{20} = -\frac{m_1}{m_2} \vec{v}_{10}$$

$$\vec{v}_1 - \vec{V} = \vec{v}_{10}$$

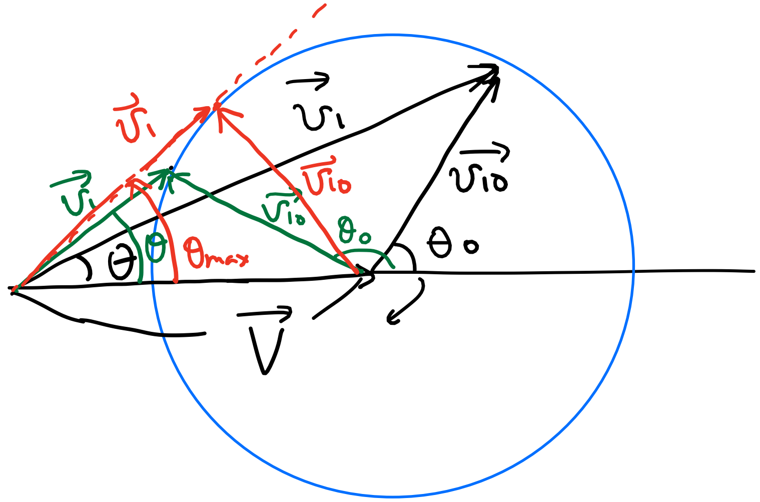


$$|\vec{V}| < v_{10}$$

$$0 \leq \theta_0 \leq \pi$$

$$\downarrow$$

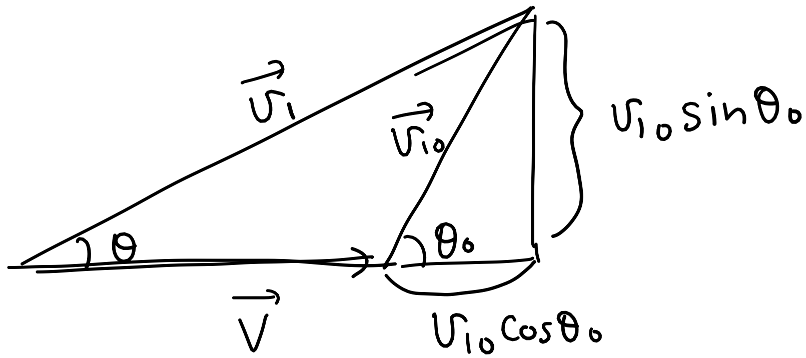
$$0 \leq \theta \leq \pi$$



$$|\vec{V}| > v_{10}$$

$$\sin\theta_{max} = \frac{v_{10}}{V}$$

$$0 \leq \theta \leq \theta_{max}$$



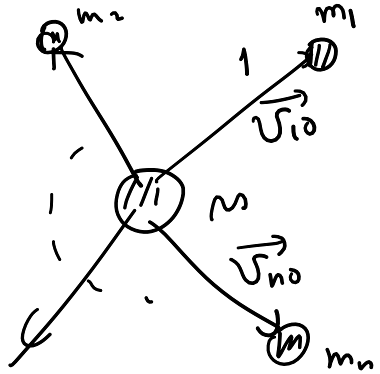
$$\tan\theta = \frac{v_{10}\sin\theta_0}{V + v_{10}\cos\theta_0} = \frac{\sin\theta_0}{\underbrace{\left(\frac{V}{v_{10}}\right)}_{\gamma} + \cos\theta_0}$$

$$\gamma > 1$$

$$\gamma < 1 \qquad \gamma + \cos\theta_0 = 0 \quad \theta_0 \text{ exist}$$

② $1 \rightarrow n$ 개.

CM.



$$\vec{p} = 0 = \sum_{j=1}^{n} m_j \vec{v}_{j0}$$

$$\rightarrow = \sum_{j=2}^{n} m_j \vec{v}_{j0} + m_1 \vec{v}_{10}$$

$$\vec{v}_{10} = -\frac{1}{m_1} \sum_{j=2}^{n} m_j \vec{v}_{j0}$$

$$\epsilon = \underset{\text{internal}}{E_i} - \sum_{j=1}^{n} E_{ji} = \underbrace{\sum_{j=2}^{n} \frac{1}{2} m_j v_{j0}^2} + \frac{1}{2} m_1 v_{10}^2$$

ϵ_{max} for fixed v_{10} ; max of $\sum_{j=2}^{n} \frac{1}{2} m_j v_{j0}^2$

with fixed $\sum_{j=2}^{n} m_j \vec{v}_{j0}$

(ex)

$$f(x, y) = x^2 + y^2 \quad \text{with} \quad x + y = 1 \leftarrow g(x, y) = 0$$

Lagrange's Undetermined Multiplier (라그랑쥐 미정계수法)

$$F(x, y) = f(x, y) + \lambda g(x, y)$$ → treat x, y as indpendent

$$\frac{\partial F}{\partial x} = 0 \qquad \frac{\partial F}{\partial y} = 0$$

$$F(\vec{v}_{j0}) = \sum_{j=2}^{n} \frac{1}{2} m_j \vec{v}_{j0} \cdot \vec{v}_{j0} + \vec{\lambda} \cdot \sum_{j=2}^{n} m_j \vec{v}_{j0}$$

$$\frac{\partial F}{\partial \vec{v}_{j0}} = m_j \vec{v}_{j0} - m_j \vec{\lambda} = 0 \xrightarrow{j=2,\cdots,n} \vec{v}_{j0} = \vec{\lambda}$$

$$\vec{v}_{10} = -\frac{1}{m_1} \underbrace{\left(\sum_{j=2}^{n} m_j \right)}_{= M - m_1} \vec{\lambda} \rightarrow \vec{\lambda} = \underbrace{-\frac{m_1}{M-m_1} \vec{v}_{10}}_{= \vec{v}_{j0}}$$

$$\epsilon_{max} = \sum_{j=2}^{n} \frac{1}{2} m_j v_{j0}^2 + \frac{1}{2} m_1 v_{10}^2 = \sum_{j=2}^{n} \frac{1}{2} m_j \left(\frac{m_1}{M-m_1} \right)^2 v_{10}^2 + \frac{1}{2} m_1 v_{10}^2$$

$$= \frac{1}{2} \left(\frac{m_1}{\cancel{M-m_1}} \right)^2 v_{10}^2 \underbrace{\left(\sum_{j=2}^{n} m_j \right)}_{= \cancel{M-m_1}} + \underbrace{\frac{1}{2} m_1 v_{10}^2}_{T_{10,max}}$$

$$= \frac{1}{\cancel{2}} \frac{m_1}{M-m_1} \underbrace{v_{10}^2 \cdot m_1}_{\cancel{2} T_{10,max}} = T_{10,max} \underbrace{\left(1 + \frac{m_1}{M-m_1} \right)}_{\frac{M}{M-m_1}}$$

$$\therefore \quad T_{10,max} = \frac{M-m_1}{M} \epsilon_{max}$$

//

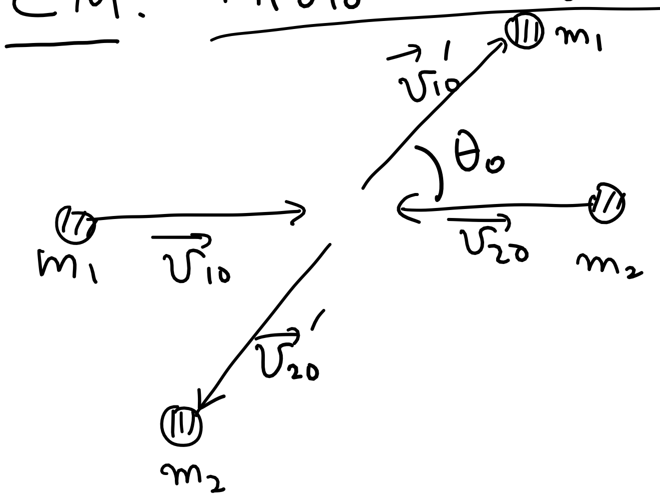
Collision

$2 \longrightarrow 2$

CM.

$$m_1 \vec{v}_{10} + m_2 \vec{v}_{20} = 0 \quad , \quad m_1 \vec{v}_{10}' + m_2 \vec{v}_{20}' = 0 \quad \text{always}$$

$$\vec{v}_{20} = -\frac{m_1}{m_2}\vec{v}_{10} \qquad \vec{v}_{20}' = -\frac{m_1}{m_2}\vec{v}_{10}'$$



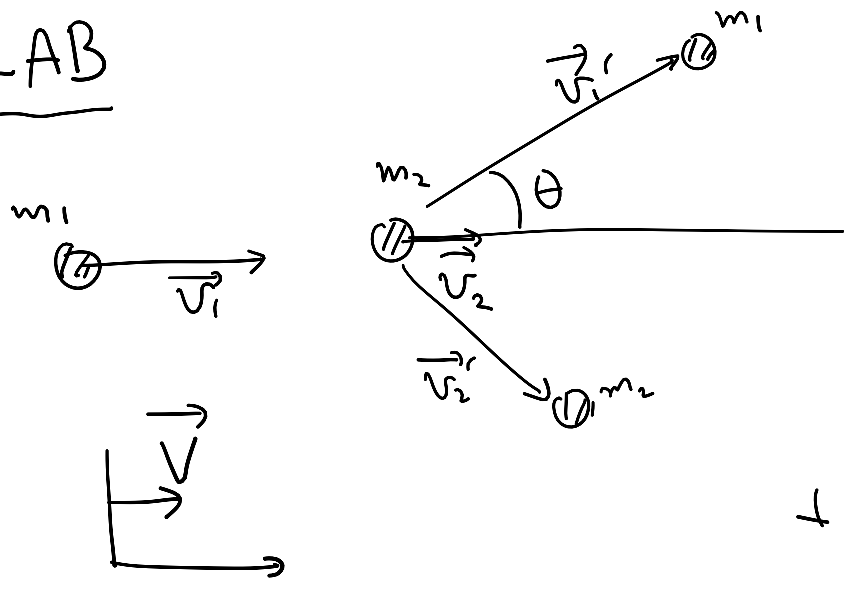
$$E_{1i} + \frac{1}{2} m_1 v_{10}^2 + E_{2i} + \frac{1}{2} m_2 v_{20}^2$$

$$= E_{1i}' + \frac{1}{2} m_1 v_{10}'^2 + E_{2i}' + \frac{1}{2} m_2 v_{20}'^2$$

$$E_{1i} + E_{2i} \neq E_{1i}' + E_{2i}' \quad \text{(in general inelastic)}$$

$$\text{"} \quad = \quad \text{"} \quad \text{(special elastic)}$$

LAB



$$\vec{v}_i - \vec{V} = \vec{v}_{i0} \qquad i = 1, 2$$

$$\vec{v}_i' - \vec{V} = \vec{v}_{i0}'$$

$$m_1 (\vec{v}_1 - \vec{V}) = \vec{v}_{10}\, m_1$$

$$+ \big) \; m_2 (\vec{v}_2 - \vec{V}) = \vec{v}_{20}\, m_2$$

$$= 0 \longrightarrow \vec{V} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

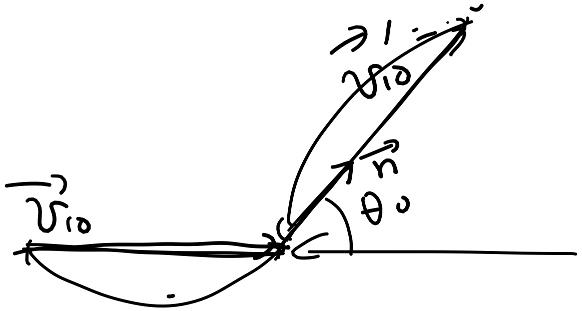
$$\vec{v}_1 - \vec{V} = \vec{v}_{10} = \vec{v}_1 - \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \frac{m_2(\vec{v}_1 - \vec{v}_2)}{m_1+m_2} = \frac{m_2\vec{v}}{m_1+m_2}$$

$$\vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}$$

$$\vec{v}_1 - \vec{v}_2 \equiv \vec{v}$$

$$\vec{v}_2 - \vec{V} = \vec{v}_{20} = \vec{v}_2 - \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \frac{m_1(\vec{v}_2 - \vec{v}_1)}{m_1+m_2} = -\frac{m_1\vec{v}}{m_1+m_2}$$

CM.



$$\frac{m_1^2}{m_2^2} v_{10}^2$$

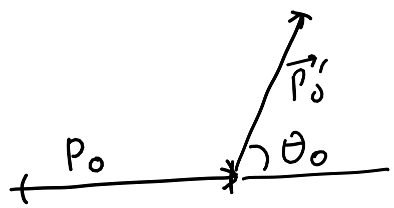
$$\vec{p}_0 \equiv m_1\vec{v}_{10}$$

$$\vec{p}_0{}' \equiv m_1\vec{v}_{10}{}'$$

$$E_{1i} + \frac{1}{2}m_1 v_{10}^2 + E_{2i} + \frac{1}{2}m_2 v_{20}^2 = E_{1i} + E_{2i} + \frac{p_0^2}{2m}$$

$$= E_{1i}' + \frac{1}{2}m_1 v_{10}'^2 + E_{2i}' + \frac{1}{2}m_2 v_{20}'^2 = E_{1i}' + E_{2i}' + \frac{p_0'^2}{2m}$$

elastic $\rightarrow \dfrac{p_0^2}{2m} = \dfrac{p_0'^2}{2m} \rightarrow p_0 = p_0' \rightarrow v_{10} = v_{10}'$



LAB.

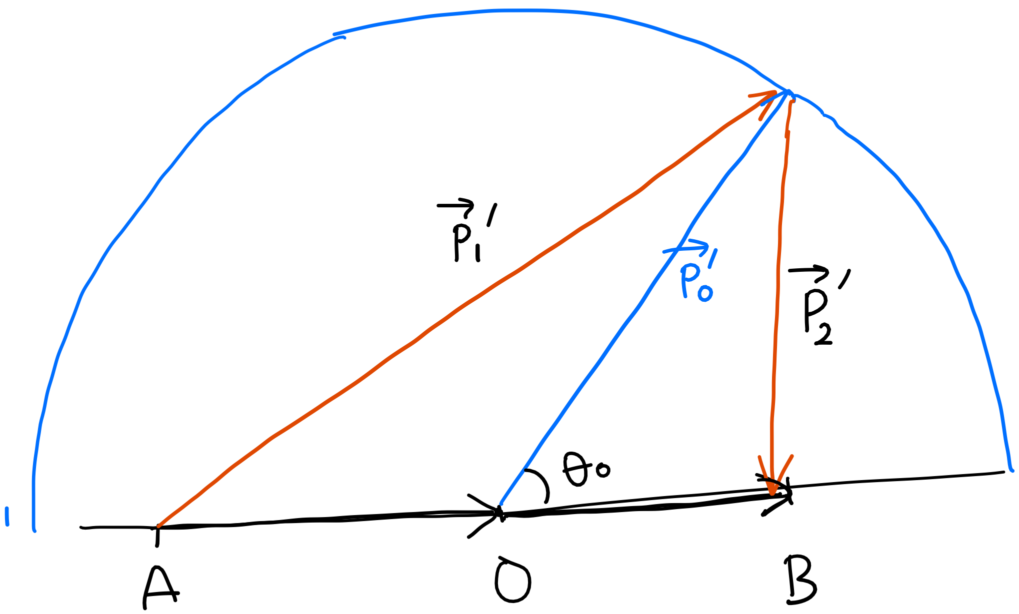
$$\underbrace{\vec{v}_1' - \vec{V} = \vec{v}_{10}'} \longrightarrow \underset{\overset{\shortparallel}{\vec{p}_1'}}{m_1 \vec{v}_1'} = \underbrace{m_1 \vec{V}}_{\vec{AO}} + \underbrace{m_1 \vec{v}_{10}'}_{\vec{p}_0'}$$

$$\left\{ \underset{\overset{\shortparallel}{\vec{p}_2'}}{m_2 \vec{v}_2'} = \underbrace{m_2 \vec{V}}_{\vec{OB}} + \underbrace{m_2 \vec{v}_{20}'}_{\overset{\shortparallel}{\underline{\vec{p}_{20}'}}} \right.$$

$$\underset{\overset{\shortparallel}{\vec{p}_0'}}{\vec{p}_{10}'} + \vec{p}_{20}' = 0 \qquad \vec{p}_{20}' = -\vec{p}_0'$$

$$m_1 \vec{V} = \frac{\overset{\vec{p}_1}{m_1 \vec{v}_1} + \overset{\vec{p}_2}{m_2 \vec{v}_2}}{m_1 + m_2} \cdot m_1 = \frac{m_1}{m_1 + m_2} (\vec{p}_1 + \vec{p}_2) = \vec{AO}$$

$$m_2 \vec{V} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} \cdot m_2 = \frac{m_2}{m_1 + m_2} (\vec{p}_1 + \vec{p}_2) = \vec{OB}$$



	before	after
LAB	$\vec{v}_2$	$\vec{v}_2'$
CM	$\vec{v}_{20}$	$\vec{v}_{20}'$

$$\boxed{\vec{v}_2 = 0} \;\Rightarrow\; \vec{p}_2 = 0$$

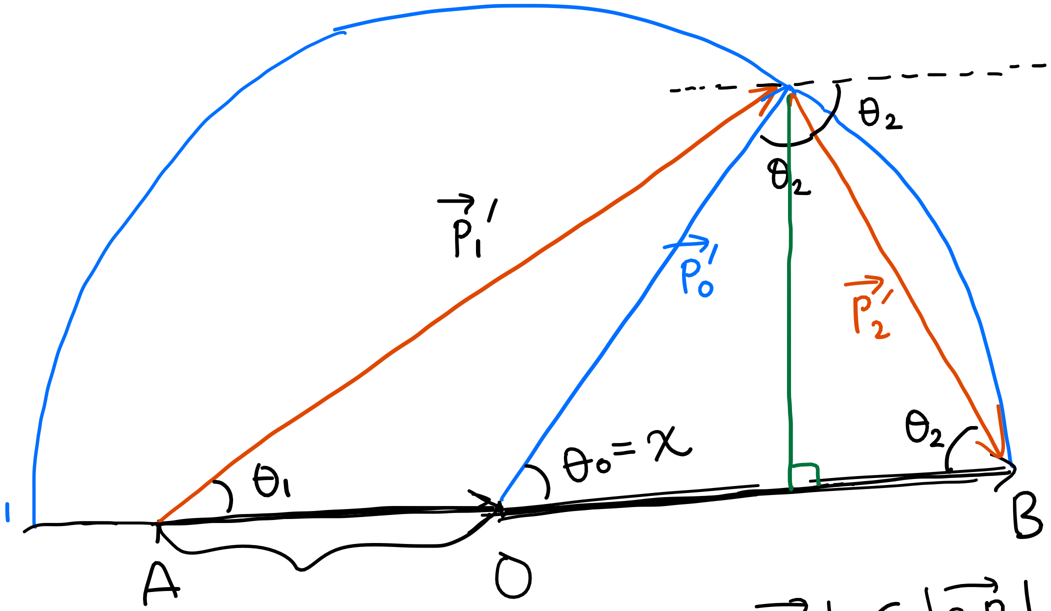
$\vec{v}_1$ m_2 $v_2 = 0$

$$\vec{OB} = \frac{m_2}{m_1+m_2}\vec{p}_1 = \frac{m_1 m_2}{m_1+m_2}\vec{v}_1 = m\vec{v}$$

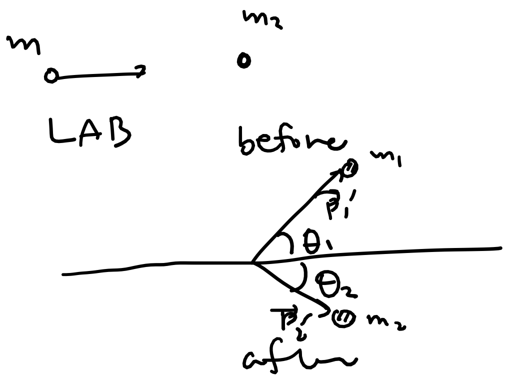
$$\vec{p}_1 = m_1\vec{v}_1, \quad \frac{m_1 m_2}{m_1+m_2} = m, \quad \vec{v}_1 = \vec{v} \equiv \vec{v}_1 - \vec{v}_2$$

$$= \vec{p}_0 = m_1\vec{v}_{10} = m\vec{v}.$$

$$|\vec{p}_0| = |\vec{p}_0{}'| = |\vec{OB}|$$



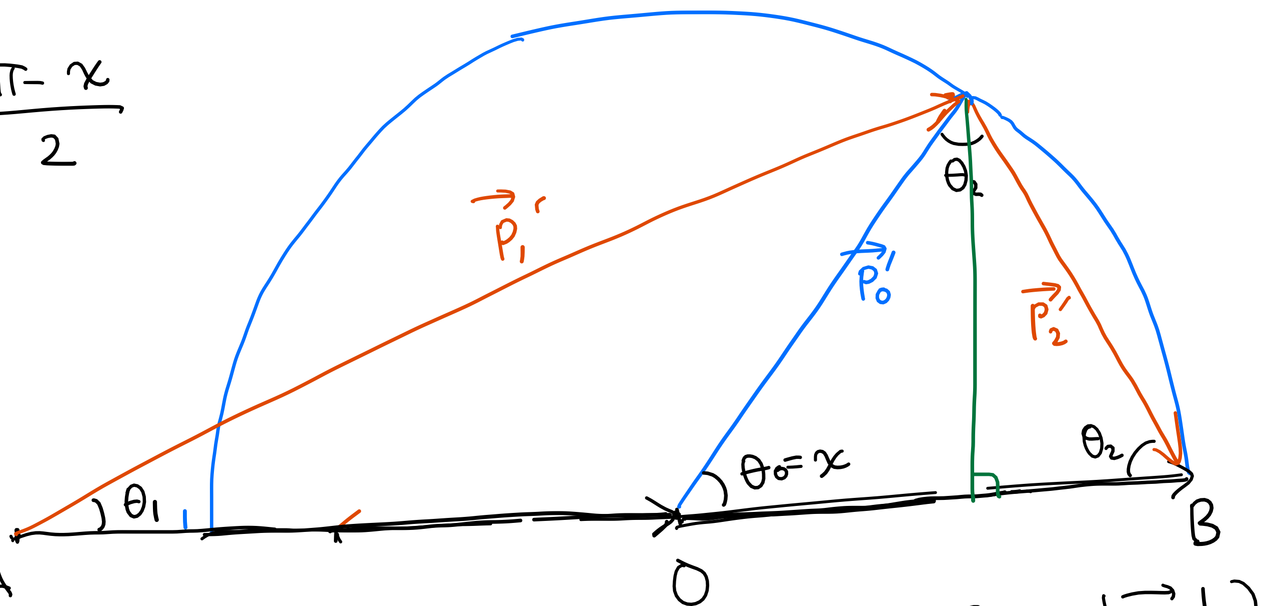
$m_1 < m_2$ $\quad$ ($|\vec{AO}| < |\vec{OB}|$)



$$2\theta_2 + \chi = \pi$$

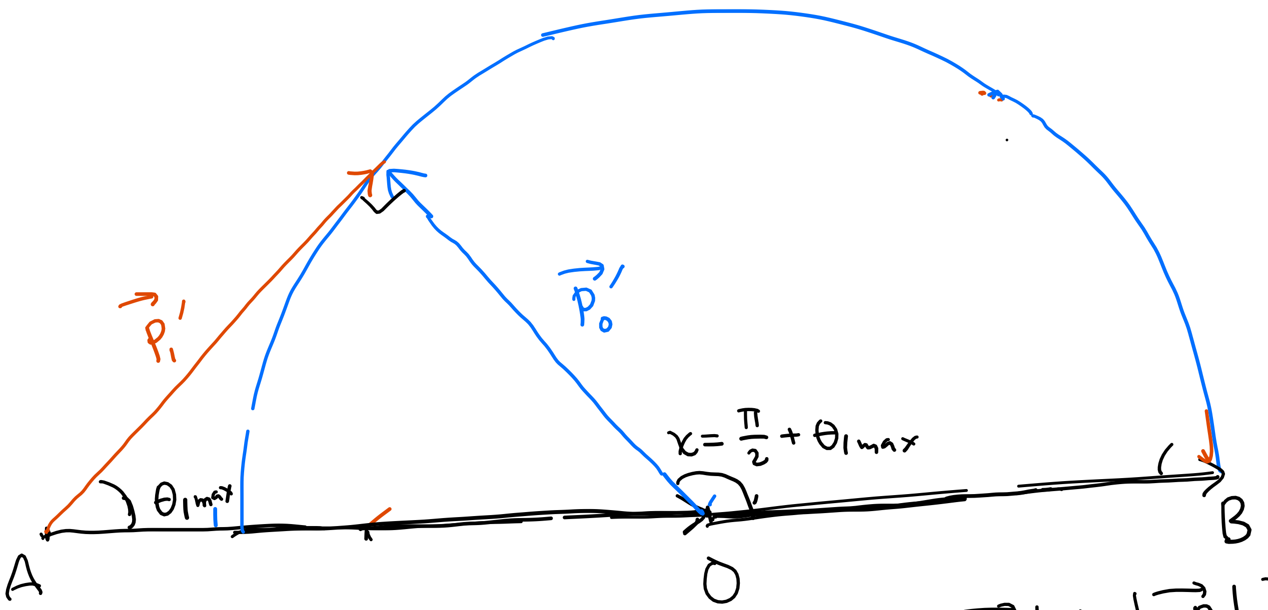
$$\Downarrow$$

$$\theta_2 = \frac{\pi - \chi}{2}$$



$m_1 > m_2$ $\quad$ ($|\vec{AO}| > |\vec{OB}|$)

θ_1: maximum $(m_1 > m_2)$

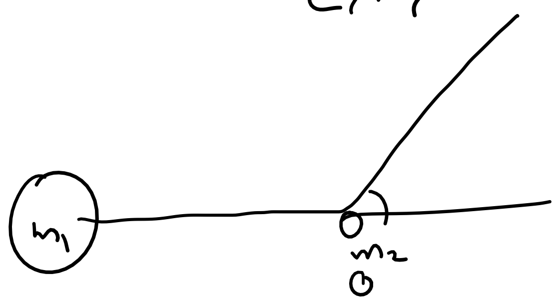


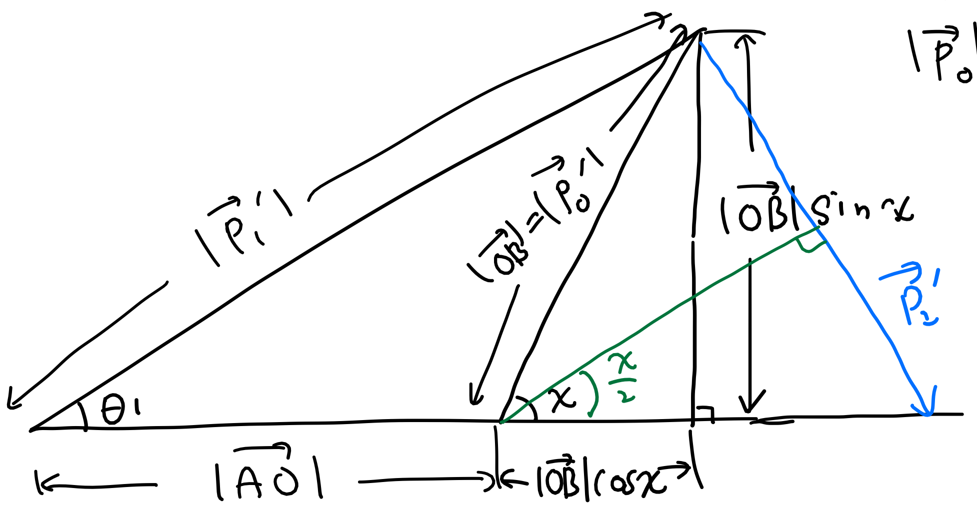
$m_1 > m_2$ $\quad (|\vec{AO}| > |\vec{OB}|)$

$\hookrightarrow$ θ_1 has maximum

$$\sin\theta_{1max} = \frac{|\vec{p}_0{}'|}{|\vec{AO}|} = \frac{|\vec{OB}|}{|\vec{AO}|} = \frac{m_2}{m_1}$$

LAB





$$\underset{\substack{\| \\ mv}}{|\vec{p}_0|} = |\vec{p}_0{}'| = |\vec{OB}| \equiv b$$

$|\vec{OB}| = b$

$|\vec{AO}| = a$

$$\frac{a}{b} = \frac{m_1}{m_2}$$

$$\left\{ \begin{array}{l} \tan\theta_1 = \dfrac{|\vec{OB}|\sin x}{|\vec{AO}| + |\vec{OB}|\cos x} = \dfrac{\sin x}{\underbrace{\frac{|\vec{AO}|}{|\vec{OB}|}}_{\frac{m_1}{m_2}} + \cos x} = \dfrac{\sin x}{\frac{m_1}{m_2} + \cos x} \\ \\ \theta_2 = \dfrac{\pi - x}{2} \end{array} \right.$$

if $\frac{m_1}{m_2} = 1 \to \theta_1 = \frac{x}{2}$

$$p_1' = |\vec{p}_1{}'| = m_1 v_1' = \sqrt{(a + b\cos x)^2 + (b\sin x)^2}$$

$$= \sqrt{a^2 + b^2 + 2ab\cos x} = b\sqrt{1 + \left(\frac{m_1}{m_2}\right)^2 + 2\frac{m_1}{m_2}\cos x}$$

$$m \equiv \frac{m_1 m_2}{m_1 + m_2}$$

$$\therefore \quad v_1' = \frac{mv}{m_1}\sqrt{1 + \left(\frac{m_1}{m_2}\right)^2 + 2\frac{m_1}{m_2}\cos x} = \frac{m_2}{m_1 + m_2} v \sqrt{1 + \left(\frac{m_1}{m_2}\right)^2 + 2\frac{m_1}{m_2}\cos x}$$

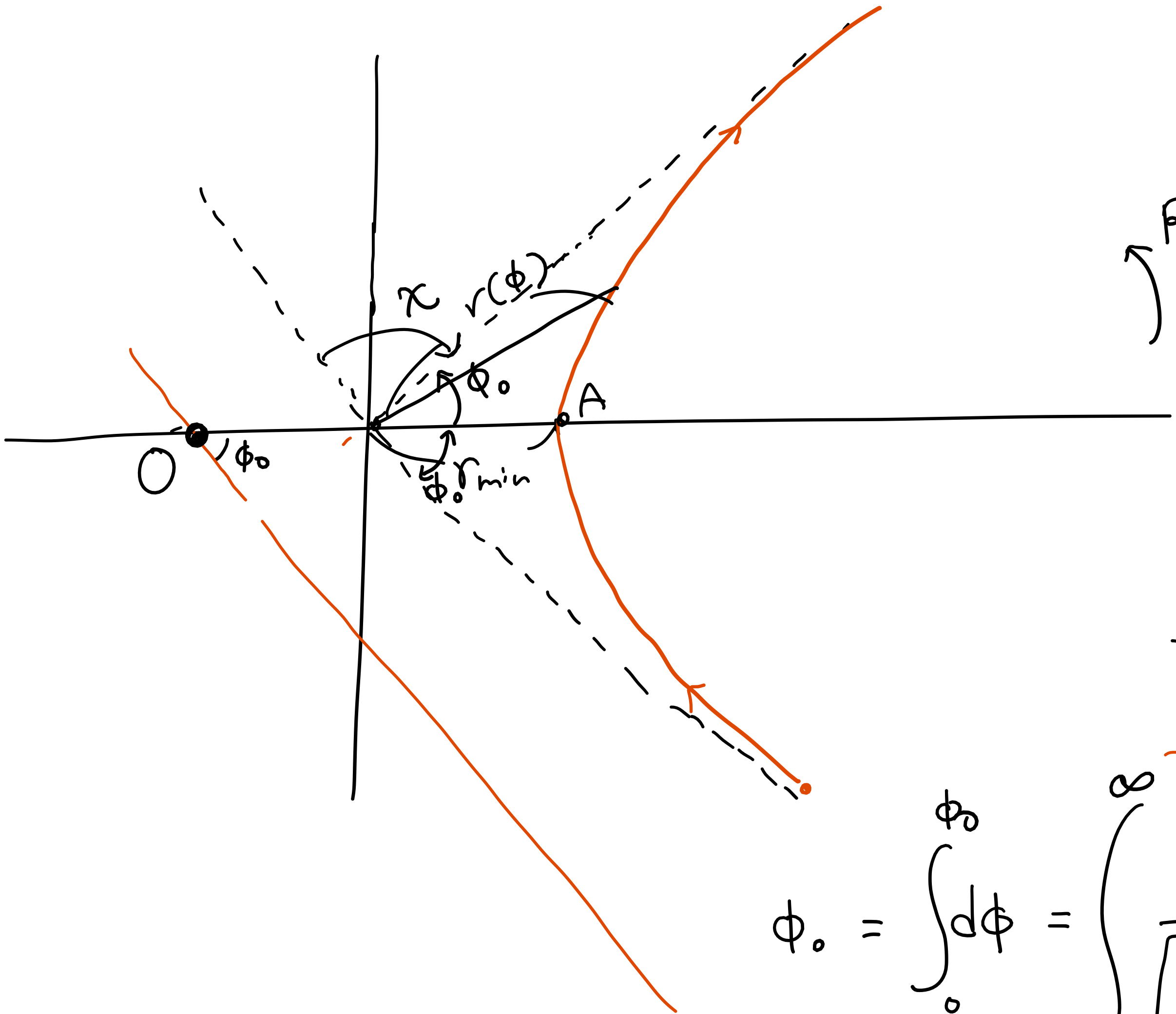
$$p_2' = m_2 v_2' = 2|\overrightarrow{OB}| \sin\frac{\chi}{2} = 2 m v \sin\frac{\chi}{2}$$

$$\therefore \quad v_2' = 2\frac{m}{m_2} v \sin\frac{\chi}{2} = \frac{2 m_1 v}{m_1 + m_2} \sin\frac{\chi}{2} \; /\!/$$

H.W. Problem on p. 47

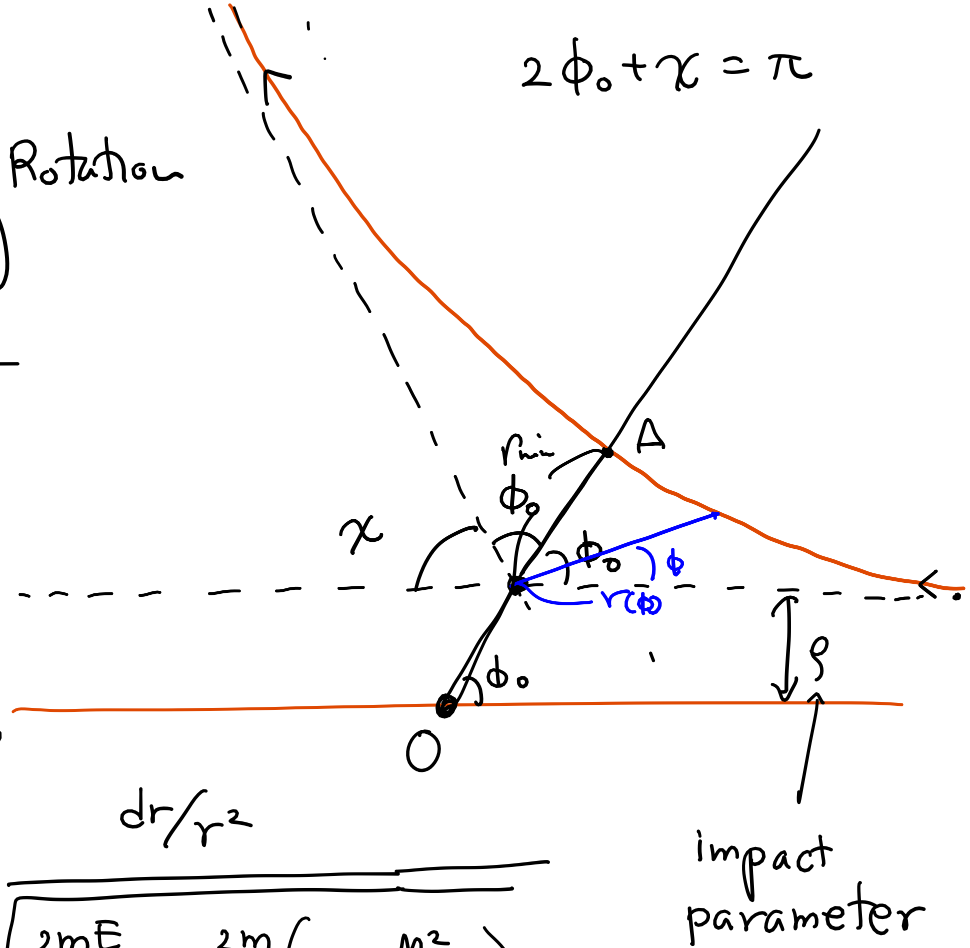
18. Scattering

$U = -\frac{\alpha}{r}$ $\quad (\alpha < 0)$ $\quad$ (Rutherford scattering)



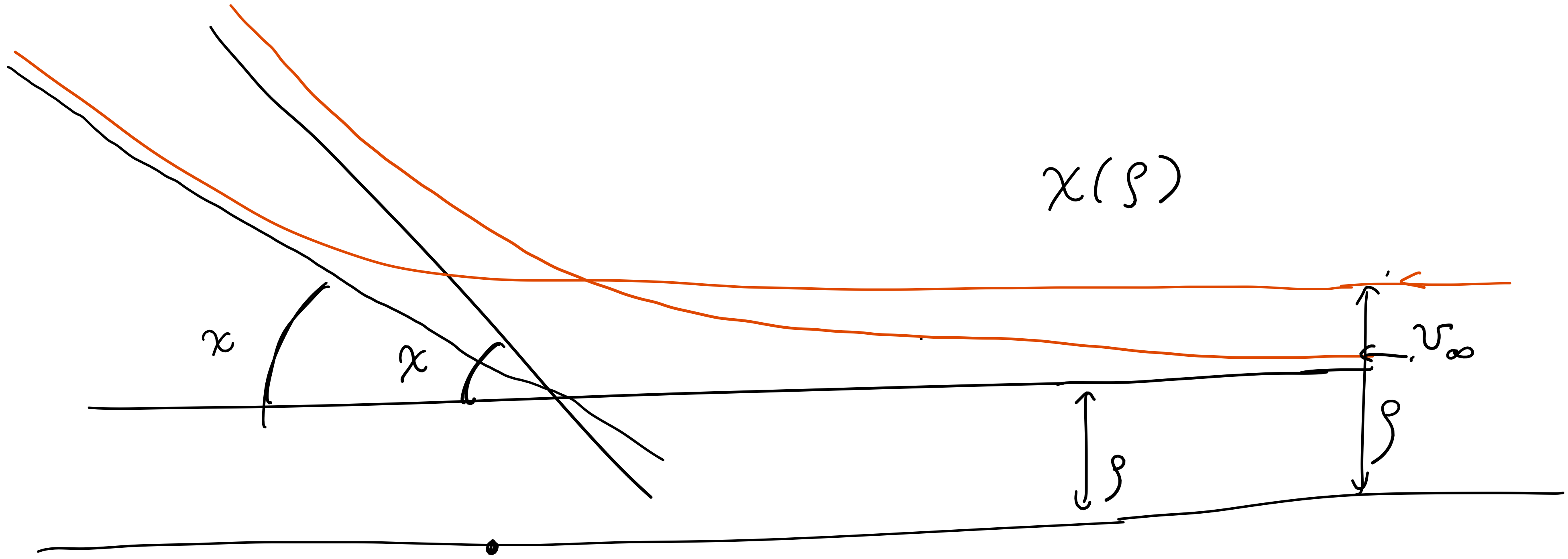
Rotation

$$2\phi_0 + \chi = \pi$$



$$\phi_0 = \int_0^{\phi_0} d\phi = \int_{r_{min}}^{\infty} \frac{dr/r^2}{\sqrt{\frac{2mE}{M^2} - \frac{2m}{M^2}\left(U + \frac{M^2}{2mr^2}\right)}}$$

$\chi(\rho)$



$$M = \rho\, m v_\infty$$

$$E = \underset{\underset{0}{\shortparallel}}{U_{eff}} + \frac{1}{2} m v_\infty^2 = \frac{1}{2} m v_\infty^2$$

at $r = \infty$

$$\frac{2mE}{M^2} = \frac{2\,\frac{1}{2} m^2 \cancel{v_\infty^2}}{\rho^2 m^2 \cancel{v_\infty^2}} = \frac{1}{\rho^2}$$

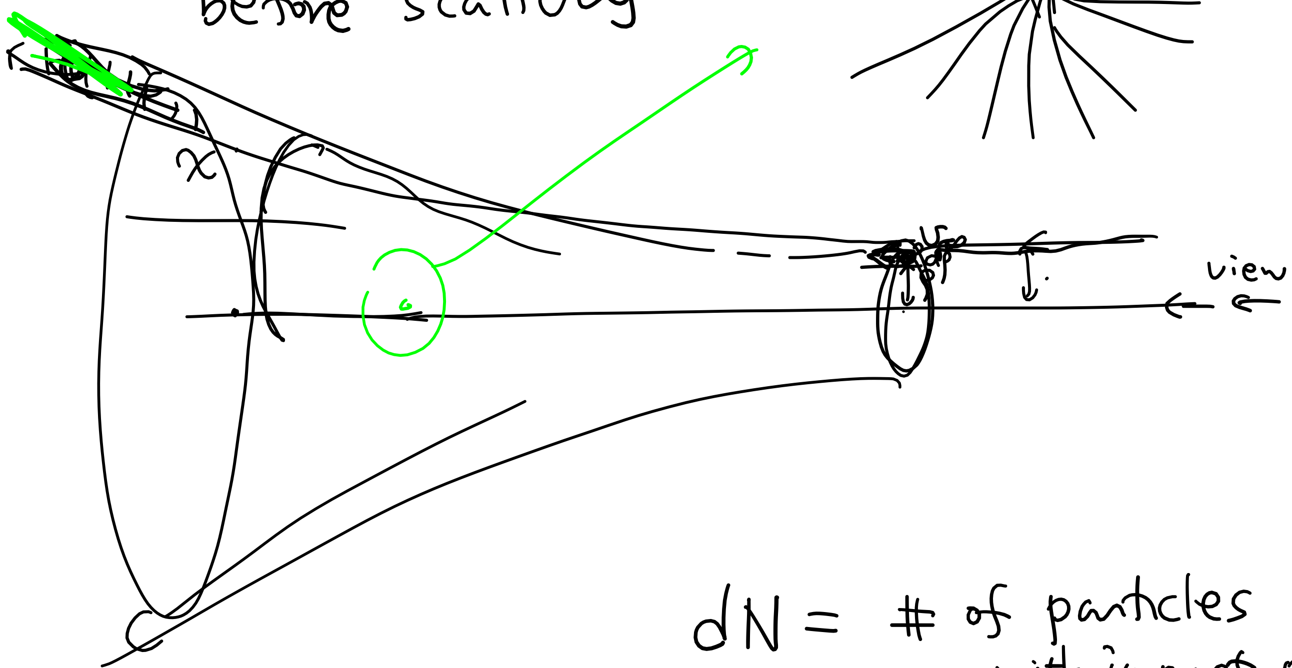
$$\frac{2m}{M^2} = \frac{2m}{\rho^2 m^2 v_\infty^2} = \frac{2}{\rho^2 m v_\infty^2}$$

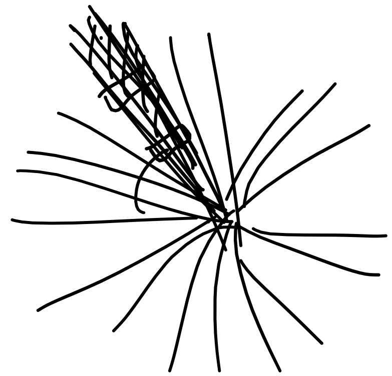
$$\phi_0 = \int_0^{\phi_0} d\phi = \int_{r_{min}}^{\infty} \frac{dr/r^2}{\sqrt{\underbrace{\frac{2mE}{M^2}}_{\downarrow\ \frac{1}{\rho^2}} - \underbrace{\frac{2m}{M^2}\left(U + \frac{M^2}{2mr^2}\right)}_{\frac{2mU}{M^2} + \frac{1}{r^2}}}} = \int_{r_{min}}^{\infty} \frac{dr/r^2}{\sqrt{\frac{1}{\rho^2} - \frac{2U}{\rho^2 m v_\infty^2} - \frac{1}{r^2}}} = F(\rho)$$

$$\phi_0 = \frac{\pi - \chi}{2}$$

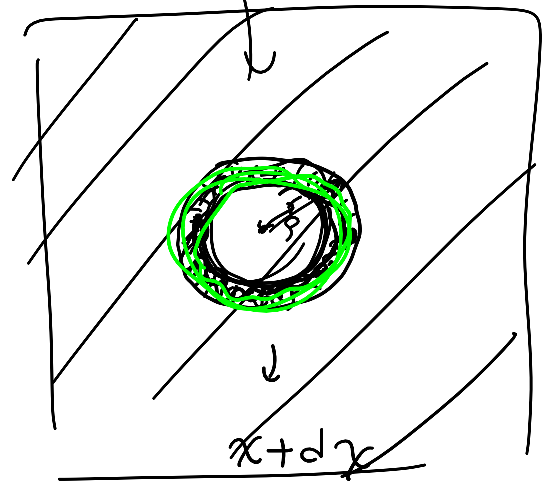
cross section

before scattering





Uniform with n



dN = # of particles before scattering with impact parameter between ρ & $\rho + d\rho$ $= n 2\pi \rho \, d\rho$

$=$ # of scattered particles between χ & $\chi + d\chi$

n = particles per unit area

$$\therefore \quad dN = n \, 2\pi\rho \, d\rho$$

$$\therefore \quad \boxed{d\sigma = 2\pi \rho \, d\rho}$$

$$\frac{dN}{n} \equiv d\sigma$$

$=$ [area] → cross section

$$d\sigma = 2\pi\rho\, d\rho$$

$$\downarrow$$

$$\frac{d\sigma}{d\chi} = 2\pi\rho\left|\frac{d\rho}{d\chi}\right| = f(\chi)$$

differential cross section (미분 산란 단면적)

$\underbrace{\text{between } \chi \text{ \& } \chi + d\chi : \# \text{ of scattered particles}}_{dN/n}$ per incident density n per $d\chi$

§19. Rutherford's formula

$$U = \frac{\alpha}{r}$$

$$u \equiv \frac{1}{r} \rightarrow du = -\frac{1}{r^2}dr \qquad du = -A\sin\psi\, d\psi$$

$$\frac{\pi-\chi}{2} = \int_{r_{min}}^{\infty} \frac{dr/r^2}{\sqrt{\frac{1}{\rho^2} - \frac{2\alpha}{\rho^2 m v_\infty^2 r} - \frac{1}{r^2}}} = \int_0^{u_{max}} \frac{du}{\sqrt{\underbrace{\frac{1}{\rho^2} - \frac{2\alpha}{\rho^2 m v_\infty^2}u - u^2}}} = -\int_{\psi_1}^{0} d\psi$$

$$\frac{\alpha}{\rho^2 m v_\infty^2} = A\cos\psi_1$$

$$= \psi_1 = \cos^{-1}\left(\frac{\alpha}{\rho^2 m v_\infty^2 A}\right)$$

$$\left\{ \underbrace{\left(\frac{\alpha}{\rho^2 m v_\infty^2}\right)^2 + \frac{1}{\rho^2}}_{A^2} - \underbrace{\left(u + \frac{\alpha}{\rho^2 m v_\infty^2}\right)^2}_{=(A\cos\psi)^2} \right.$$

$$\frac{\pi-\chi}{2} = \psi_1 = \cos^{-1}\left(\frac{\alpha}{\rho^2 m v_\infty^2 A}\right) \qquad \sqrt{\left(\frac{\alpha}{\rho^2 m v_\infty^2}\right)^2 + \frac{1}{\rho^2}} \equiv A$$

$$= \cos^{-1}\left(\frac{\alpha}{\rho^{\cancel{2}} m v_\infty^2} \frac{1}{\frac{1}{\cancel{\rho}}\sqrt{1+\left(\frac{\alpha}{\rho m v_\infty^2}\right)^2}}\right)$$

$$\frac{\pi-\chi}{2} = \cos^{-1}\left(\frac{\alpha/\rho m v_\infty^2}{\sqrt{1+\left(\frac{\alpha}{\rho m v_\infty^2}\right)^2}}\right)$$

$$\cos\left(\frac{\pi-\chi}{2}\right) = \frac{\alpha/\rho m v_\infty^2}{\sqrt{1+\left(\frac{\alpha}{\rho m v_\infty^2}\right)^2}} = \underset{\overset{\shortparallel}{s}}{\sin\frac{\chi}{2}} = \frac{\beta}{\sqrt{1+\beta^2}}$$

$\alpha/\rho m v_\infty^2 \equiv \beta$

$$s^2 = \frac{\beta^2}{1+\beta^2}$$

$$\frac{1}{\beta^2} = \cot^2\frac{\chi}{2} \leftarrow \beta^2 = \tan^2\frac{\chi}{2} \leftarrow \beta^2\underbrace{(1-s^2)}_{=c^2} = s^2$$

$$= \left(\frac{\rho m v_\infty^2}{\alpha}\right)^2 \longrightarrow \therefore \rho^2 = \frac{\alpha^2}{m^2 v_\infty^4}\cot^2\frac{\chi}{2}$$

$$\frac{d\sigma}{d\chi} = 2\pi\left|\rho\frac{d\rho}{d\chi}\right|$$

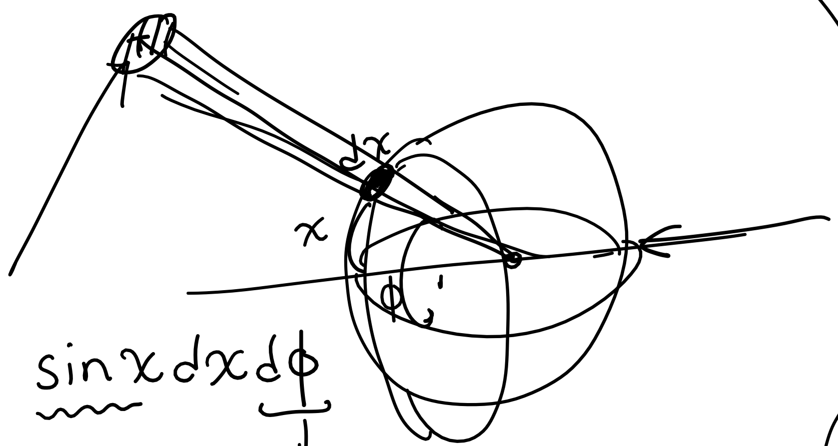
$$\rho^2 = \frac{\alpha^2}{m^2 v_\infty^4}\cot^2\frac{\chi}{2}$$

$$\underbrace{2\rho\frac{d\rho}{d\chi}} = \text{"}\ \cancel{2}\cot\frac{\chi}{2}\cdot\frac{1}{\cancel{2}}\frac{(-1)}{\sin^2\frac{\chi}{2}}$$

$$\to \therefore \frac{d\sigma}{d\chi} = \underbrace{2\pi}_{\int d\phi}\frac{1}{2}\frac{\alpha^2}{m^2 v_\infty^4}\frac{\cos\frac{\chi}{2}}{\sin^3\frac{\chi}{2}}$$

$$0 \le \chi \le \pi$$

solid angle

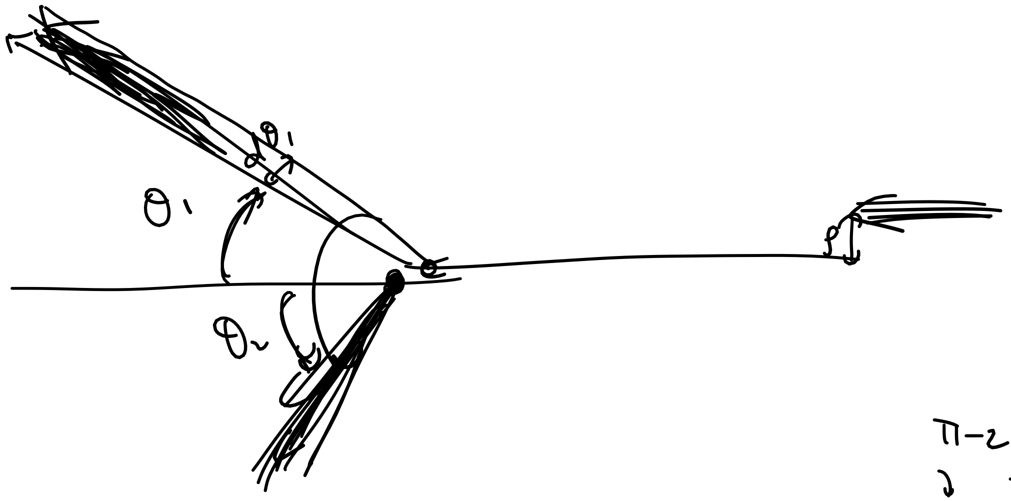


$$d\Omega = \sin\chi\, d\chi\, \underbrace{d\phi}_{2\pi}$$

$$\frac{d\sigma}{d\chi} = \int_0^{2\pi}\underbrace{\left(\frac{d\sigma}{d\Omega}\right)}d\phi \longrightarrow \frac{d\sigma}{d\Omega} = \frac{1}{2\pi}\frac{d\sigma}{d\chi}\frac{1}{\sin\chi}$$

$$\longrightarrow \left(\frac{\alpha}{2mv_\infty^2}\right)^2\frac{1}{\sin^4\frac{\chi}{2}} = \frac{1}{2}\frac{\alpha^2}{m^2 v_\infty^4}\frac{\cos\frac{\chi}{2}}{\sin^3\frac{\chi}{2}}\cdot\frac{1}{\sin\chi}$$

LAB.



$$\chi = \pi - 2\theta_2$$

$$d\sigma_2 = d\sigma = \underbrace{2\pi}_{\int d\phi_2} \cancel{\tfrac{1}{2}} \frac{\alpha^2}{m^2 v_\infty^4} \frac{\cos\frac{\chi}{2}}{\sin^3\frac{\chi}{2}} \, d\chi \qquad \left(\chi = \pi - 2\theta_2\right)$$

$$= \qquad \text{"} \qquad \frac{\sin\theta_2}{\cos^3\theta_2} \cancel{2} \, d\theta_2$$

$$d\Omega_2 = \sin\theta_2 \, d\theta_2 \, d\phi_2$$

$$\boxed{\frac{d\sigma_2}{d\Omega_2} = \frac{\alpha^2}{m^2 v_\infty^4} \underbrace{\frac{1}{\cos^3\theta_2}}}$$

If $m_2 \gg m_1 \to \underline{\theta_1 \approx \chi}$, $\underline{m \approx m_1}$ $\qquad m = \frac{m_1 m_2}{m_1 + m_2} \approx \frac{m_1 \cancel{m_2}}{\cancel{m_2}}$

$$d\sigma_1 = d\sigma = 2\pi \, \frac{1}{2} \left(\frac{\alpha^2}{m^2 v_\infty^4}\right) \frac{\cos\theta_1/2}{\sin^3\frac{\theta_1}{2}} \, d\theta_1$$

$$d\Omega_1 = \sin\theta_1 \, d\theta_1 \, d\phi_1$$

$$\to \frac{d\sigma_1}{d\Omega_1} = \underline{\frac{1}{4} \overbrace{\left(\frac{\alpha^2}{m^2 v_\infty^4}\right)} \frac{1}{\sin^4\frac{\theta_1}{2}}}$$

$$\left(\frac{\alpha}{4E_1}\right)^2 = \left(\frac{\alpha}{4 \cdot \underbrace{\tfrac{1}{2} m_1 v_\infty^2}_{E_1}}\right)^2$$

if $m_1 = m_2$ $\quad m = \frac{m_1}{2}$ $\longrightarrow$ $\theta_1 = \frac{x}{2}$ (cf) $\theta_2 = \frac{\pi - x}{2}$ $\longrightarrow$ $\theta_1 + \theta_2 = \frac{\pi}{2}$

$$d\sigma_1 = d\sigma = \underbrace{2\pi}_{\int d\phi_1} \cancel{\frac{1}{2}} \frac{\alpha^2}{m^2 v_\infty^4} \frac{\cos\theta_1}{\sin^3\theta_1} 2\, d\theta_1$$

$$\| $$

$$d\sigma_2$$

$$d\Omega_1 = \sin\theta_1 \, d\theta_1 \, d\phi_1$$

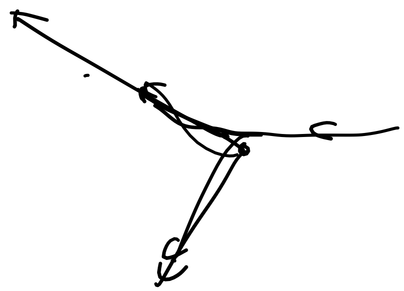
$$\frac{d\sigma_1}{d\Omega_1} = \cancel{\frac{1}{2}} \frac{\alpha^2}{\left(\frac{m_1}{2}\right)^2 v_\infty^4} \frac{\cos\theta_1}{\sin^4\theta_1} \cancel{2} = \left(\frac{\alpha}{\frac{m_1}{2} v_\infty^2} \right)^2 \frac{\cos\theta_1}{\sin^4\theta_1}$$

$$= \left(\frac{\alpha}{E_1} \right)^2 \frac{\cos\theta_1}{\sin^4\theta_1} \quad ✓$$

× $d\Omega_2 = \sin\theta_2 \, d\theta_2 \, d\phi_2 = \cos\theta_1 \, d\theta_1 \, d\phi_1$

if $m_1 = m_2$, identical

$d\sigma_1$ $\quad \theta_1 \to \frac{\pi}{2} - \theta_2 \to$ $d\sigma_2$



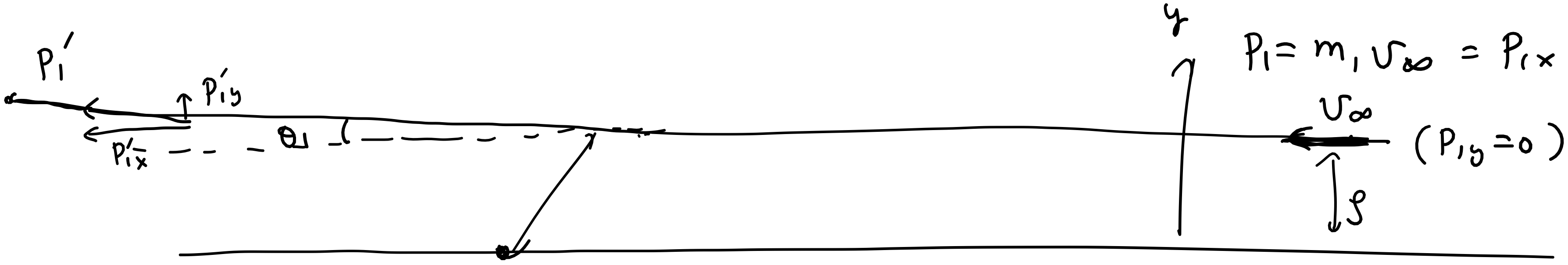
$$+ \left(\frac{\alpha}{E_1} \right)^2 \frac{1}{\cos^3\theta_1}$$

$$\Longrightarrow \quad \frac{d\sigma}{d\Omega} = \left(\frac{\alpha}{E_1} \right)^2 \left(\frac{1}{\sin^4\theta} + \frac{1}{\cos^4\theta} \right) \cos\theta \quad \leftarrow$$

§ 20. small angle

$\underline{U \ll 1}$

LAB



$$\theta_1 \approx \tan\theta_1 = \frac{p_{1y}'}{p_{1x}'} \approx \frac{p_{1y}'}{p_1 = m_1 v_\infty} \qquad p_{1y}' = \int_{-\infty}^{\infty} F_y \, dt$$

$$\vec{F} = -\vec{\nabla} U \rightarrow F_y = -\frac{\partial U(r)}{\partial y} = -\frac{dU}{dr}\frac{\partial r}{\partial y} = -\frac{y}{r}\frac{dU}{dr}$$

$$r^2 = x^2 + y^2 \xrightarrow{\frac{\partial}{\partial y}} 2r\frac{\partial r}{\partial y} = 2y \rightarrow \frac{\partial r}{\partial y} = \frac{y}{r}$$

$y \approx \rho$

$$\therefore \quad p_{1y}' = \int \left(-\frac{y}{r}\frac{dU}{dr}\right) \underset{\frac{dx}{v_\infty}}{dt} \simeq -\frac{\rho}{v_\infty}\int_{-\infty}^{\infty} \frac{1}{r}\frac{dU}{dr}\, dx$$

$$r^2 = x^2 + \rho^2 \rightarrow x = \sqrt{r^2 - \rho^2} \qquad dx = \frac{r\,dr}{\sqrt{r^2 - \rho^2}} \qquad \frac{dU}{dr}\frac{dr}{\sqrt{r^2 - \rho^2}}$$

$$\theta_1 = -\frac{2\rho}{m_1 v_\infty^2}\int_\rho^\infty \frac{dU}{dr}\frac{dr}{\sqrt{r^2-\rho^2}} \Rightarrow \frac{d\sigma_1}{d\theta_1} = \frac{d\sigma}{d\theta_1} = 2\pi\rho\left|\frac{d\rho}{d\theta_1}\right|$$

$$= F(\rho)$$

$$d\sigma_1 = \frac{\rho}{\sin\theta_1}\left|\frac{d\rho}{d\theta_1}\right| d\Omega_1$$

$$\overset{\theta_1 \ll 1}{\approx} \frac{\rho}{\theta_1}\left|\frac{d\rho}{d\theta_1}\right| d\Omega_1$$

Prob 1. (18.4) $\rightarrow$

$$\phi_0 = \int_{r_{min}}^\infty \frac{\rho/r^2\, dr}{\sqrt{1-\frac{\rho^2}{r^2}-\frac{2U(r)}{mv_\infty^2}}} = -\frac{\partial}{\partial\rho}\int_{r_{min}}^{R\to\infty} \underbrace{\sqrt{\underbrace{1-\frac{\rho^2}{r^2}}_{a} - \underbrace{\frac{2U}{mv_\infty^2}}_{\epsilon}}}\, dr$$

$$\sqrt{a-\epsilon} = \sqrt{a}\sqrt{1-\frac{\epsilon}{a}} \simeq \sqrt{a}\left(1-\frac{\epsilon}{2a}\right)$$

$$\sqrt{1-\frac{\rho^2}{r^2}} - \frac{\cancel{2}U/mv_\infty^2}{\cancel{2}\sqrt{1-\frac{\rho^2}{r^2}}}$$

$$= \underbrace{\int_\rho^\infty \frac{\rho/r^2}{\sqrt{1-\frac{\rho^2}{r^2}}}\,dr}_{\frac{\pi}{2}} + \frac{\partial}{\partial\rho}\int_\rho^\infty \frac{U/mv_\infty^2}{\sqrt{1-\frac{\rho^2}{r^2}}}\,dr \rightarrow \phi_0 = \frac{\pi}{2} + \frac{\partial}{\partial\rho}\int_\rho^\infty \frac{U/mv_\infty^2}{\sqrt{1-\frac{\rho^2}{r^2}}}\,dr$$

$$\phi_0 - \frac{\pi}{2} = \frac{\partial}{\partial\rho}\int_\rho^\infty \frac{U(r)/mv_\infty^2}{\sqrt{1-\frac{\rho^2}{r^2}}}\,dr = \frac{1}{mv_\infty^2}\frac{\partial}{\partial\rho}\underbrace{\int_\rho^\infty \frac{1}{2}\frac{U}{\sqrt{r^2-\rho^2}}\,d(r^2)}$$

$r^2 \equiv z$

$$\frac{1}{2}\int_{\rho^2}^\infty \frac{U}{\sqrt{z-\rho^2}}\,dz = \frac{1}{2}\left(\cancel{2U\sqrt{z-\rho^2}\Big|_{\rho^2}^{\infty}} - \int_{\rho^2}^\infty 2\frac{dU}{dz}\sqrt{z-\rho^2}\,dz\right)$$

$z \equiv r^2$

$\downarrow$

$dz = \cancel{2r}\,dr$

$$\frac{dU}{dz} = \frac{dU}{\cancel{2r}\,dr}$$

$$= -\frac{1}{2}\int 2\,\frac{dU}{dr}\sqrt{r^2-\rho^2}\,dr$$

$$\phi_0 - \frac{\pi}{2} = \frac{-1}{mv_\infty^2}\frac{\partial}{\partial\rho}\underbrace{\int_\rho^\infty \frac{dU}{dr}\sqrt{r^2-\rho^2}\,dr}$$

$$\frac{-\cancel{2}\rho}{\cancel{2}\sqrt{r^2-\rho^2}}$$

θ_1

$=$

$\pi - 2\phi_0$

$=$

$$-2\left(\phi_0 - \frac{\pi}{2}\right) = \frac{(-2)\rho}{mv_\infty^2}\int_\rho^\infty \frac{dU}{dr}\frac{1}{\sqrt{r^2-\rho^2}}\,dr \quad \checkmark$$

Prob. 1, 4 on P. 50. 51
Prob. 1, 2 on P. 54
$\Big\}$ H.W.

Written
1. New
1. Derivation
1. from H.W.
→ taken here in next week

→ alps.ewha.ac.kr → 지난학기 (link)

Code ← 1. MatLab ←